

CP violation in hadronic penguins at *BABAR*

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Overview

- CP violation introduction.
- Measuring $\sin 2\beta$ in $b \rightarrow c\bar{c}s$ modes such as $B^0 \rightarrow J/\psi K_S^0$.
- Motivation for measuring $\sin 2\beta_{\text{eff}}$ in $b \rightarrow q\bar{q}s$ ($q = u, d, s$) penguins.
 - $B^0 \rightarrow \eta' K^0, \phi K_S^0, \omega K_S^0, \pi^0 K_S^0$
- Previous results.
- PEP-II collider, *BABAR* detector, and dataset.
- $B^0 \rightarrow \eta' K^0$ analysis (part of my thesis).
- Recent results from other penguins.
- Discussion.

Symmetries

- Discrete transformations C , P , and T :

- Parity (P) : $(t, \mathbf{x}) \rightarrow (t, -\mathbf{x})$.
- Time-reversal (T) : $(t, \mathbf{x}) \rightarrow (-t, \mathbf{x})$.
- Charge-conjugation (C) :
particles \rightarrow anti-particles

- Strong and electromagnetic forces conserve C , P , and T .
- In 1956, the weak force was observed to violate P ; the $V - A$ form of the weak force manifestly violates C and P .
- It was thought that weak interactions conserved CP , until Cronin and Fitch observed $K_L^0 \rightarrow \pi^+ \pi^-$ in 1964.
- In '90s, B -factories built to test CKM picture of CP violation in decays of B mesons.

CKM matrix and CP violation

- Consider the coupling of quarks to charged gauge bosons in the flavor (interaction) basis:

$$\mathcal{L}_W = g_W \left(\bar{u}^L \gamma^\mu W_\mu^+ d^L + \bar{d}^L \gamma^\mu W_\mu^- u^L \right)$$

- Transform to mass basis with $u^{L'} = U^u u^L$, $d^{L'} = U^d d^L$:

$$\mathcal{L}'_W = g_W \left(\bar{u}^L \gamma^\mu W_\mu^+ V d^L + \bar{d}^L \gamma^\mu W_\mu^- V^* u^L \right)$$

where $V = (U^{u\dagger} U^d)$ is the CKM quark mixing matrix.

- CKM matrix is 3×3 unitary matrix characterized by three real parameters and one irreducible phase.
- Because of CKM phase, $V \neq V^*$ and \mathcal{L}'_W is not invariant under CP .

CKM basics

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- By convention, V_{CKM} takes down-type quarks from flavor to mass basis.
- In Wolfenstein parameterization (to order λ^3):

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

where λ is the sine of the Cabibbo angle ($\lambda = V_{us} \simeq 0.22$) and A , ρ , and η are real and of order 1.

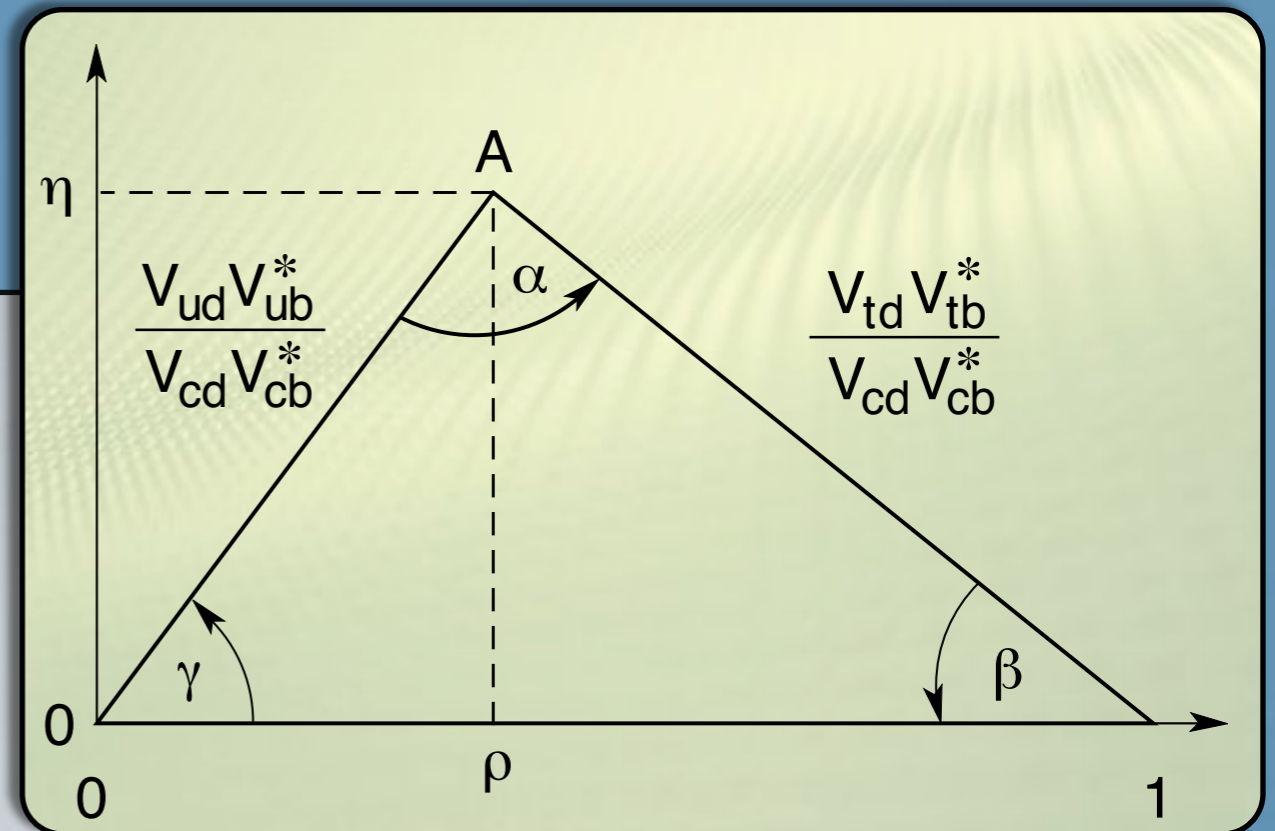
- $\eta \neq 0$ implies CP violation.

Unitarity triangle

- Unitarity of V_{CKM} yields

$$\sum_{i=u,c,t} V_{ij} V_{ik}^* = 0.$$

($j = d, s, b$; $k = d, s, b$; and $j \neq k$)



- Useful for the B system, this equation

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

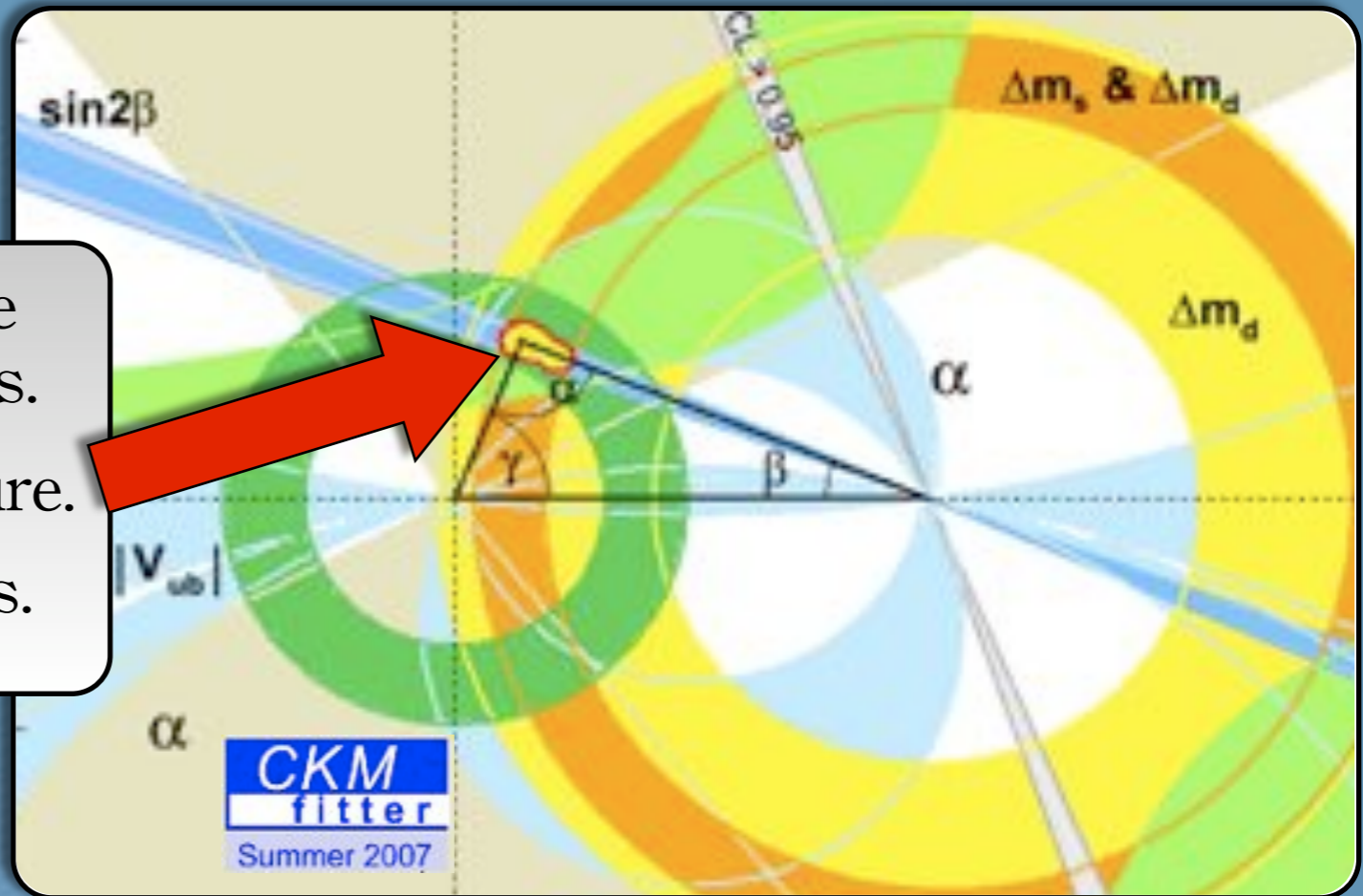
describes a triangle in complex plane of roughly equal sides.

- The apex is at $\rho + i\eta$, so that non-zero area implies CP-violation.
- The angles are defined

$$\alpha \equiv \arg \left[-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right], \quad \beta \equiv \arg \left[-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right], \quad \text{and} \quad \gamma \equiv \arg \left[-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right].$$

Overconstraining the triangle

- Measure the sides and angles of the unitary triangle in diverse processes.
- Agreement confirms the CKM picture.
- Disagreement indicates new physics.



Wassily Kandinsky
Composition VIII, 1923

$B\bar{B}$ Time Evolution

- At *BABAR*, pairs of B mesons are created in entangled states in $\Upsilon(4S)$ decays.

- Mass and flavor eigenstates differ:

$$\begin{aligned} |B_L\rangle &= p|B^0\rangle + q|\bar{B}^0\rangle \\ |B_H\rangle &= p|B^0\rangle - q|\bar{B}^0\rangle \end{aligned}$$

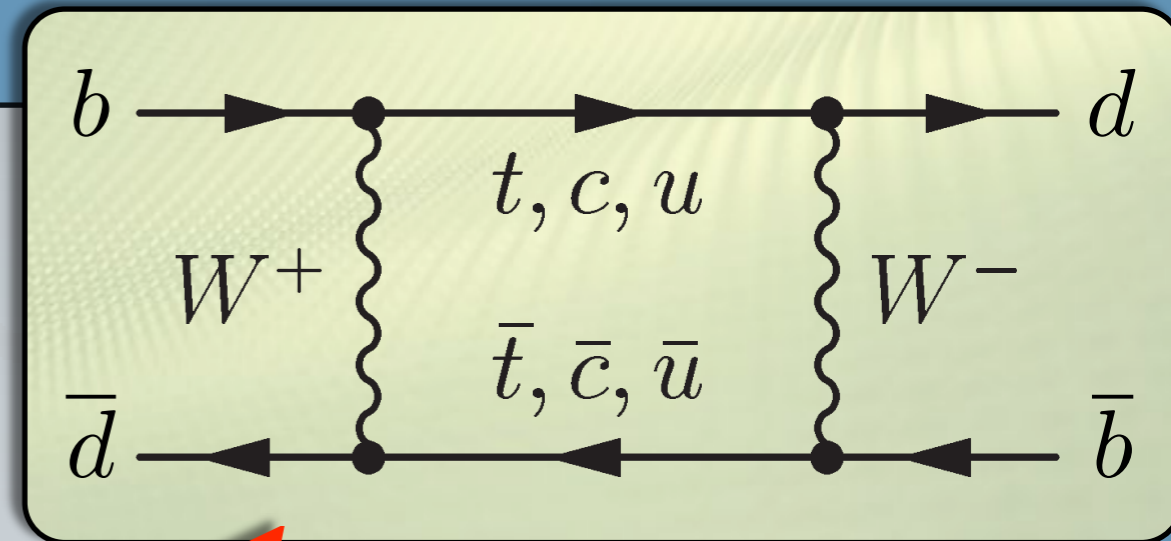
- Neutral B 's, created in flavor states, mix via second-order weak process (box diagram).

- A B^0 meson at $t = 0$ can be written at t :

$$|B_{\text{phys}}^0(t)\rangle = e^{-iMt}e^{-\Gamma t/2} \left[\cos\left(\frac{1}{2}\Delta m_B t\right) |B^0\rangle + i\frac{q}{p} \sin\left(\frac{1}{2}\Delta m_B t\right) |\bar{B}^0\rangle \right]$$

- No CP violation in mixing, q/p is pure phase.

- Note $\Delta\Gamma_B \ll \Delta m_B$, $\Delta\Gamma_K \simeq 2\Delta m_K$, $\Delta m_B = 100 \cdot \Delta m_K$.



$$\begin{aligned} \Delta m_B &\equiv m_H - m_L \\ \Delta\Gamma_B &\equiv \Gamma_H - \Gamma_L \\ M &\equiv (m_H + m_L)/2 \\ \Gamma &\simeq \Gamma_H \simeq \Gamma_L \end{aligned}$$

Mixing-induced CP violation

- Decay to CP eigenstate f_{CP} ($\psi K_S^0, \eta' K^0$) accessible from B^0 and \bar{B}^0 involves interference between amplitudes for mixing (q/p) and decay ($A_{f_{CP}}, \bar{A}_{f_{CP}}$).
- The observable time-dependent decay rate asymmetry:

$$A_{CP}(t) = \frac{\Gamma(B_{\text{phys}}^0(t) \rightarrow f_{CP}) - \Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f_{CP})}{\Gamma(B_{\text{phys}}^0(t) \rightarrow f_{CP}) + \Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f_{CP})}$$

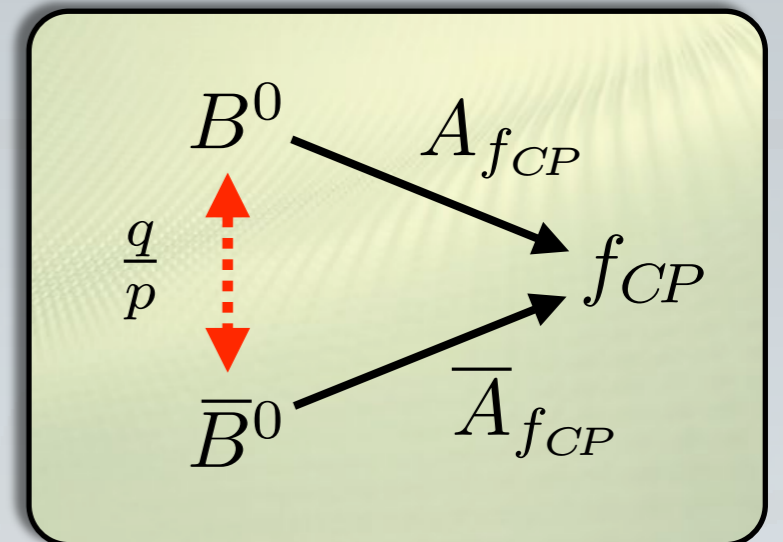
- Decay rates Γ (now called f_{\pm}) are

$$f_{\pm}(t) = \frac{e^{-t/\tau}}{4\tau} [1 \pm S_{f_{CP}} \sin(\Delta m_B t) \mp C_{f_{CP}} \cos(\Delta m_B t)]$$

- Mean B^0 lifetime is τ , the CP -violation parameters $S_{f_{CP}}$ and $C_{f_{CP}}$ are

$$S_{f_{CP}} \equiv \frac{2\text{Im}\lambda_{f_{CP}}}{1+|\lambda_{f_{CP}}|^2}, \quad C_{f_{CP}} \equiv \frac{1-|\lambda_{f_{CP}}|^2}{1+|\lambda_{f_{CP}}|^2}, \quad \lambda_{f_{CP}} \equiv \eta_{f_{CP}} \frac{q}{p} \frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}$$

- For CP conservation, $A_{CP} = S_{f_{CP}} = C_{f_{CP}} = 0$.



Experimental Aside...

- So far, $A_{CP}(t)$ and $f_{\pm}(t)$ are written in terms of the flavor at creation and the time of decay of a B meson (B_{CP}).
- We can't measure these, BUT ...
- Recall coherent B meson pairs (B_{CP} and B_{tag}).
- We can determine:
 - flavor at decay (tag) of the other B meson (B_{tag}),
 - difference between proper times of decays of B_{CP} and B_{tag} .
- We can write:

$$f_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} [1 \pm S_{f_{CP}} \sin(\Delta m_B \Delta t) \mp C_{f_{CP}} \cos(\Delta m_B \Delta t)]$$

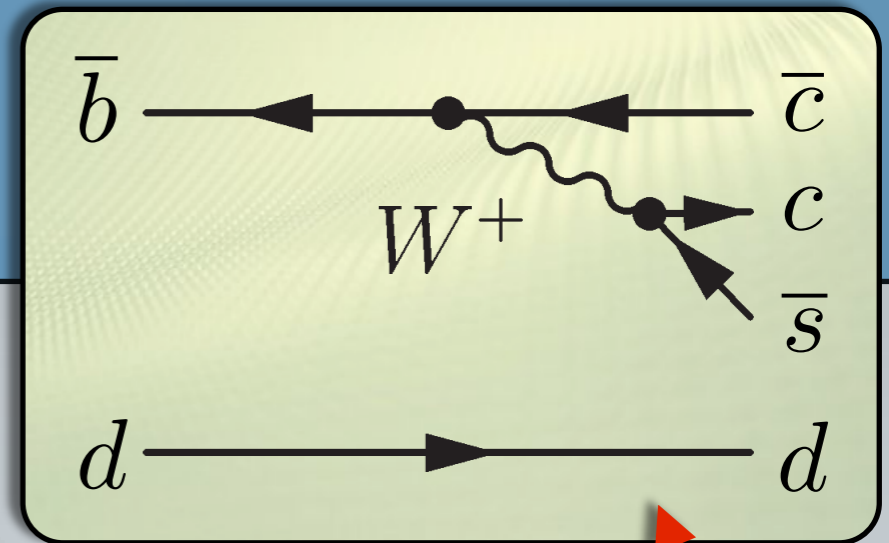
where $\Delta t \equiv t_{CP} - t_{\text{tag}}$ and the upper (lower) sign denotes a decay accompanied by a B^0 (\bar{B}^0) tag.

- We use $f_{\pm}(\Delta t)$ in the ML fit to extract $S_{f_{CP}}$ and $C_{f_{CP}}$ from the data.

$\sin 2\beta$ from $b \rightarrow c\bar{c}s$

- Examine $\lambda_{f_{CP}}$ in ``golden mode'' $B^0 \rightarrow \psi K_S^0$:

$$\lambda_{f_{CP}} \equiv \eta_{f_{CP}} \frac{q}{p} \frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}$$



$\eta_{f_{CP}}$	CP eigenvalue of f_{CP} .
$q/p \simeq V_{tb}^* V_{tb} / V_{tb} V_{td}^*$	From B mixing, independent of f_{CP} .
$A_{\psi K_S^0} \propto V_{cb} V_{cs}^*$	Single amplitude for $B^0 \rightarrow J/\psi K_S^0$.
$V_{cs} V_{cd}^* / V_{cs}^* V_{cd}$	From K mixing in the final state.

- In SM, we expect (almost exactly):

$$\lambda_{\psi K_S^0} = - \left(\frac{V_{td} V_{tb}^*}{V_{cb}^* V_{cd}} \right) \left(\frac{V_{cb} V_{cd}^*}{V_{td}^* V_{tb}} \right)$$

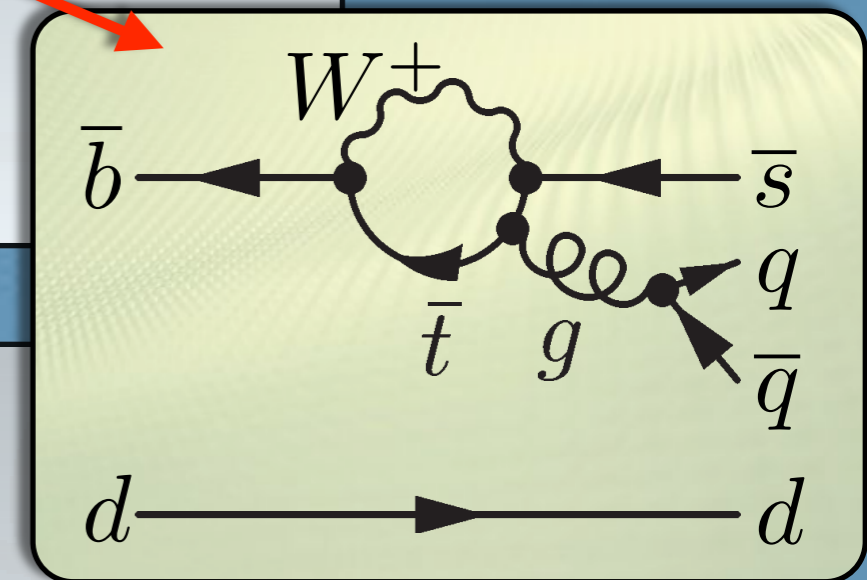
- And $S_{\psi K_S^0} = \text{Im} \lambda_{\psi K_S^0} = \sin 2\beta$ $C_{\psi K_S^0} = \frac{1 - |\lambda_{\psi K_S^0}|^2}{1 + |\lambda_{\psi K_S^0}|^2} = 0$

- No CP violation in decay because $\left| \frac{\bar{A}_{\psi K_S^0}}{A_{\psi K_S^0}} \right| = 1$; only ~ 1 amplitude contributes.

$\sin 2\beta_{\text{eff}}$ from $b \rightarrow q\bar{q}s$ penguin decays

- $b \rightarrow q\bar{q}s$ decays occur via loop processes in SM.
- $b \rightarrow s$ penguin diagrams carry same phase as $b \rightarrow c$ tree diagram.
- Penguins also sensitive to $\sin 2\beta$.

- S/C can be non-zero when two amplitudes with differing weak and strong phases contribute to the process.
- Color-suppressed tree and CKM-suppressed penguin amplitudes pollute the picture.
- $|\bar{A}_{\eta' K^0}|/|A_{\eta' K^0}| - 1 = \mathcal{O}(\lambda^2)$, so we expect:

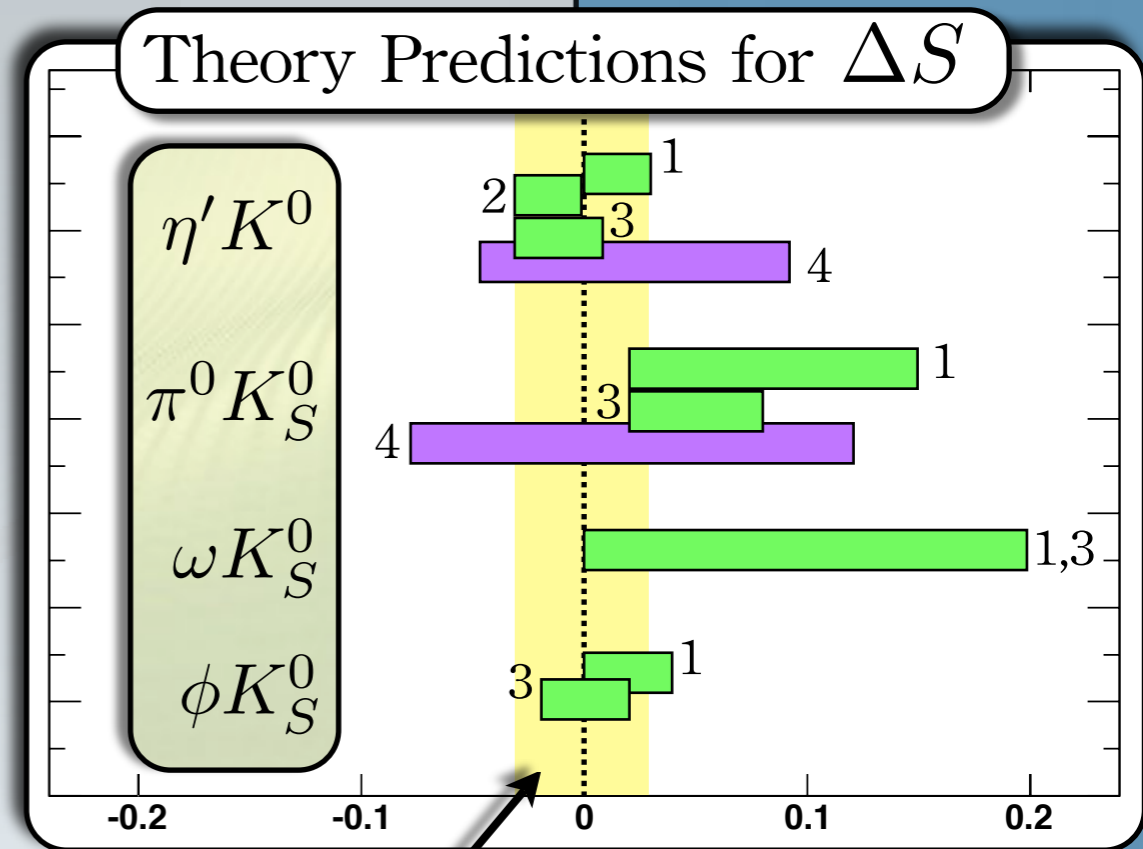
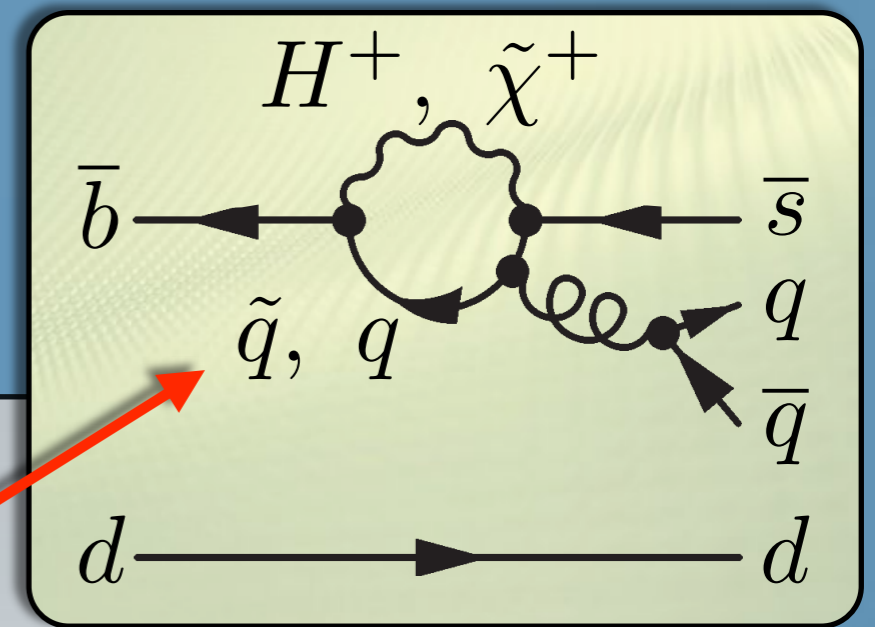


$$S_{\eta' K^0} = \sin 2\beta_{\text{eff}} \simeq \sin 2\beta$$

$$C_{\eta' K^0} \simeq 0$$

ΔS and new physics

- $\sin 2\beta_{\text{eff}}$ measured in $b \rightarrow q\bar{q}s$ penguin decays is sensitive to heavy, non-SM particles in loop.
- Deviations of ΔS ($\equiv \sin 2\beta_{\text{eff}} - \sin 2\beta$) from zero indicate new physics.
- However, even in SM, channel dependent effects cause $\Delta S \neq 0$.
- There are predictions for ΔS from
 - QCD factorization.
 - Soft collinear effective theory.
 - Flavor SU(3) symmetry.
- $\eta' K^0$ and ϕK_S^0 are theoretically cleanest modes with $\Delta S \sim 0.03$.



Yellow band is uncertainty on $\sin 2\beta$ (± 0.03).

¹QCDF Beneke, PLB620, 143 (2005)
²SCET/QCDF Williamson, Zupan, PRD74, 014003 (2006)
³QCDF Cheng, Chua, Soni, PRD72, 014006 (2005)
⁴SU(3) Gronau, Rosner, Zupan, PRD74, 093003 (2006)

Previous Results

First observation of CP violation in charmless B decays.

	$BABAR^1$	Belle ²
$S_{\eta' K^0}$	$0.58 \pm 0.10 \pm 0.03$	$0.64 \pm 0.10 \pm 0.04$
$C_{\eta' K^0}$	$-0.16 \pm 0.07 \pm 0.03$	$-0.01 \pm 0.07 \pm 0.05$
$-\eta_{\eta' K_S^0} S_{\eta' K_S^0}$	0.62 ± 0.11	0.64 ± 0.11
$C_{\eta' K_S^0}$	-0.18 ± 0.07	0.03 ± 0.07
$-\eta_{\eta' K_L^0} S_{\eta' K_L^0}$	0.32 ± 0.28	0.46 ± 0.24
$C_{\eta' K_L^0}$	-0.16 ± 0.07	-0.09 ± 0.16

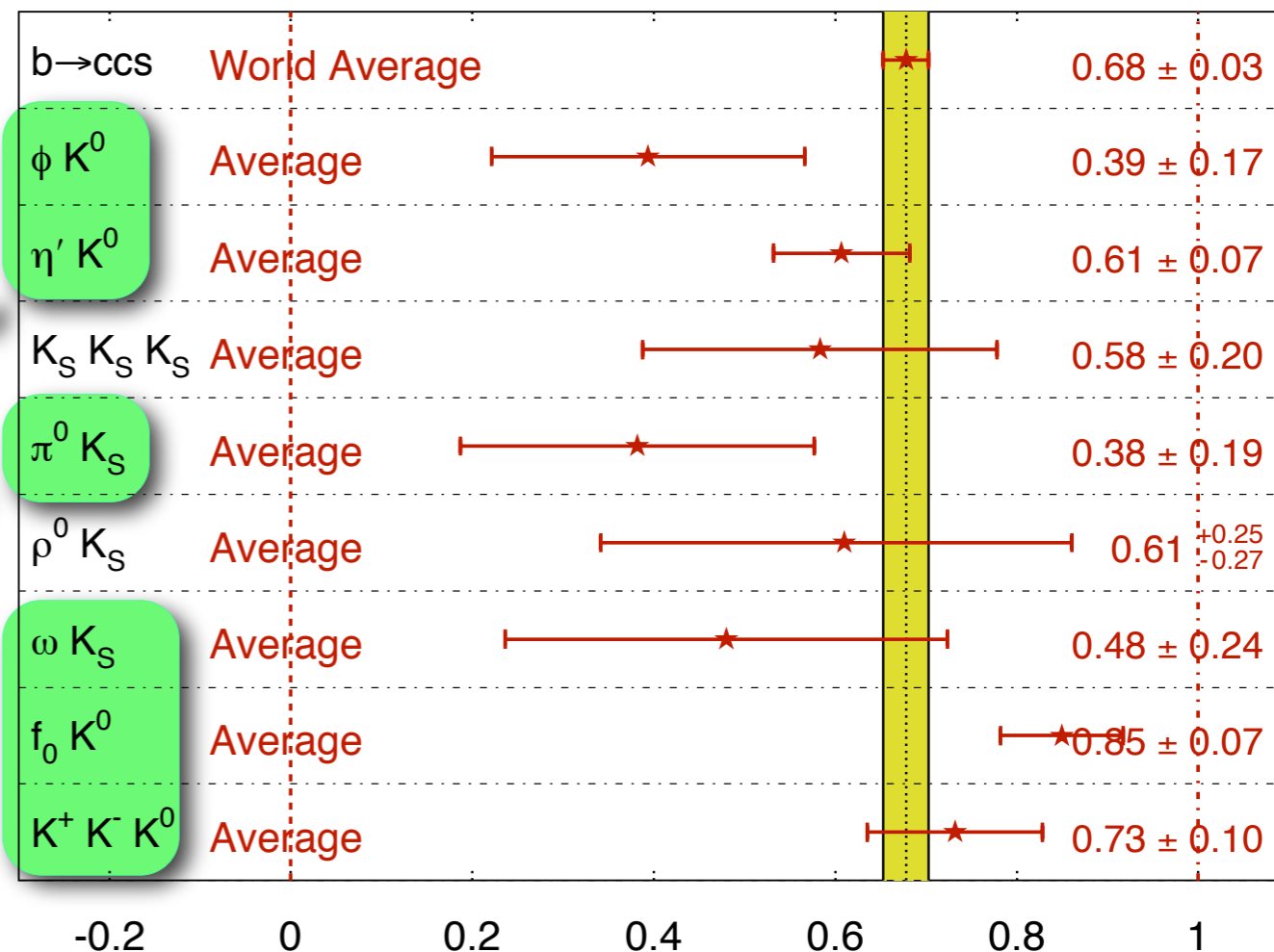
World averages (Summer '07) for $b \rightarrow s$ penguins.

$\sin 2\beta_{\text{eff}}$

HFAG
LP 2007
PRELIMINARY

- In 2005, naive average of $b \rightarrow s$ penguins differed from $b \rightarrow c\bar{c}s$ was 3.7σ .
- At last publication, discrepancy was 1.6σ (neglecting $f_0 K_S^0$).

Recent results presented today for these modes.



¹Babar Collab., PRL 98, 031801 (2007).

²Belle Collab., PRL 98 031802 (2007).

PEP-II

- Asymmetric-energy e^+e^- collider.
(9 GeV e^- , 3.1 GeV e^+)
 - Center of mass (CM) is boosted in lab ($\beta\gamma = 0.56$).
 - Separation of $B\bar{B}$ decay vertices: $20\mu\text{m} \rightarrow 200\mu\text{m}$.
-
- $\sqrt{s} \equiv E_{\text{CM}} = 10.58 \text{ GeV}$
(Mass of $\Upsilon(4S)$)
 - $\mathcal{B}(\Upsilon(4S) \rightarrow B\bar{B}) \simeq 100\%$
 - Max luminosity four times that of design:
 $1.2 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

SLAC

Linac

Aerial photo mosaic courtesy of Steve Williams.
(www.pam-rc.org)

End Station A:
partons via DIS

SPEAR:
 J/ψ , τ lepton

SLD: Precision
Z studies

BABAR

BABAR Detector

Silicon Vertex Tracker

5-layer SVT measures track impact parameters.

Drift Chamber (DCH)

- 40-layer, small-cell.
- Measures p_T .
- PID for tracks with $p_T < 0.7$ GeV from dE/dx .

DIRC

144 silica bars measure angle of Cherenkov cone, primary PID for tracks with $p_T > 0.7$ GeV.

ElectroMagnetic Calorimeter (EMC)

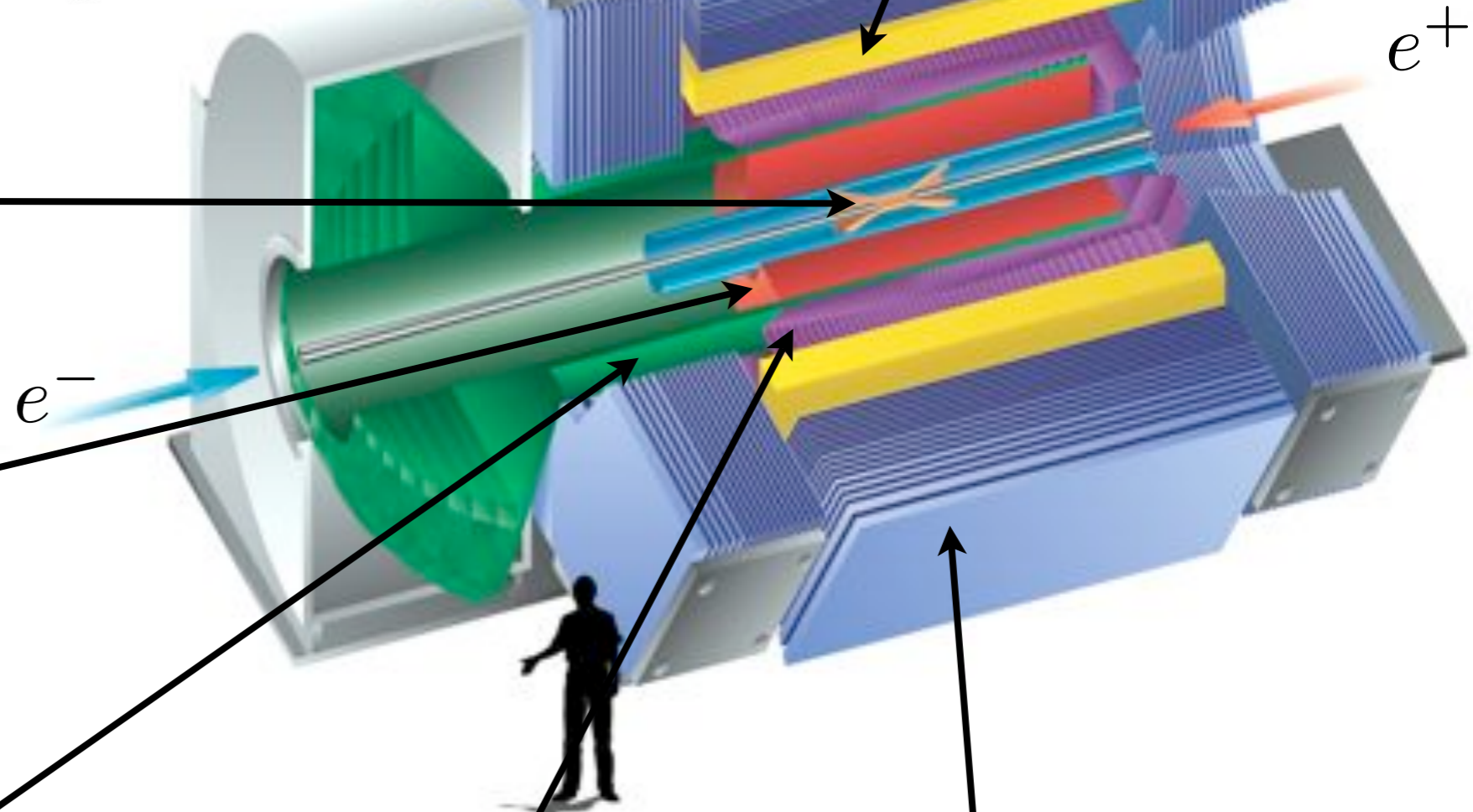
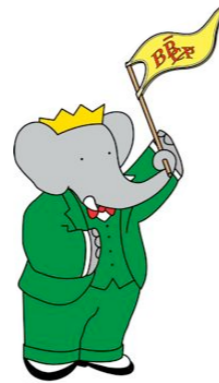
56 rings of CsI crystals. Detects energy from photons, electrons, and K_L^0 .

Instrumented Flux Return (IFR)

Resistive plate chambers and limited streamer tubes detect muons and K_L^0 .

Magnet

1.5 Tesla, superconducting magnet.

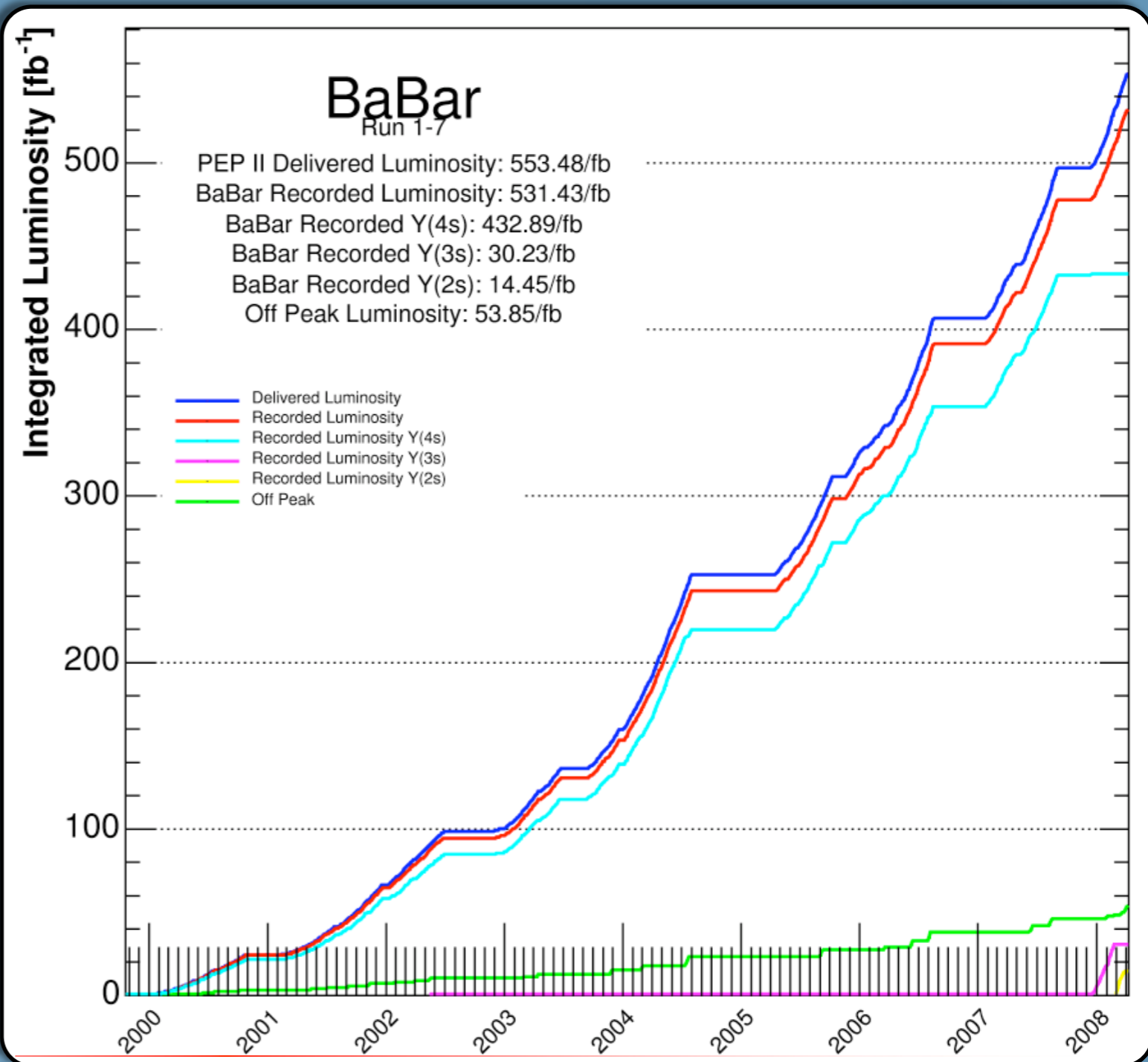


Data & Monte Carlo

- Data collected 1999-2008.
- 426 fb^{-1} on-resonance.
($\sqrt{s} = 10.58 \text{ GeV}$)
- 54 fb^{-1} off-resonance.
- $B\bar{B}$ production cross-section of $\sim 1.1 \text{ nb}$ yields 467 million $B\bar{B}$ pairs.

Large samples of GEANT4 MC

- Exclusive samples for signals and backgrounds from B decays (0.2 -1.0 million events each).
- Inclusive sample of generic $B\bar{B}$ decays (1.3 billion events).



$B^0 \rightarrow \eta' K^0$ Analysis

- Measure S and C using seven sub-decay modes.
- Crosscheck with $\eta'_{\rho\gamma} K^+$ and $\eta'_{\eta\pi\pi} K^+$.

Decay Name Key

$$\begin{aligned}\eta'_{\eta\pi\pi} &: \eta' \rightarrow \eta\pi^+\pi^-, \eta \rightarrow \gamma\gamma \\ \eta'_{\rho\gamma} &: \eta' \rightarrow \rho\gamma \\ \eta'_{5\pi} &: \eta' \rightarrow \eta\pi^+\pi^-, \eta \rightarrow \pi^+\pi^-\pi^0 \\ K_S^0 &: K_S^0 \rightarrow \pi^+\pi^- \\ K_{S00}^0 &: K_S^0 \rightarrow \pi^0\pi^0\end{aligned}$$

Mode	BF (10^{-6})	ϵ (%)	# Evts Selected	Expected Yield
$\eta'_{\eta\pi\pi} K_S^0$	3.9	27	1556	500
$\eta'_{\rho\gamma} K_S^0$	6.6	29	23905	900
$\eta'_{\eta\pi\pi} K_{S00}^0$	1.8	14	1175	110
$\eta'_{\rho\gamma} K_{S00}^0$	3.0	15	28871	200
$\eta'_{5\pi} K_S^0$	2.3	17	546	180
$\eta'_{\eta\pi\pi} K_L^0$	5.7	18	14125	450
$\eta'_{5\pi} K_L^0$	3.3	11	4951	170
$\eta' K^0$	64.9 ± 3.1	—	—	2500

Analysis Overview

1. Reconstruction and Selection:

- In kinematic fit, reconstruct B candidates from all combinations of tracks and photons.
- Apply loose selection criteria, which leave < 1.3 candidates per event.
- Select best B candidate based on B vertex probability.

2. Maximum likelihood fit:

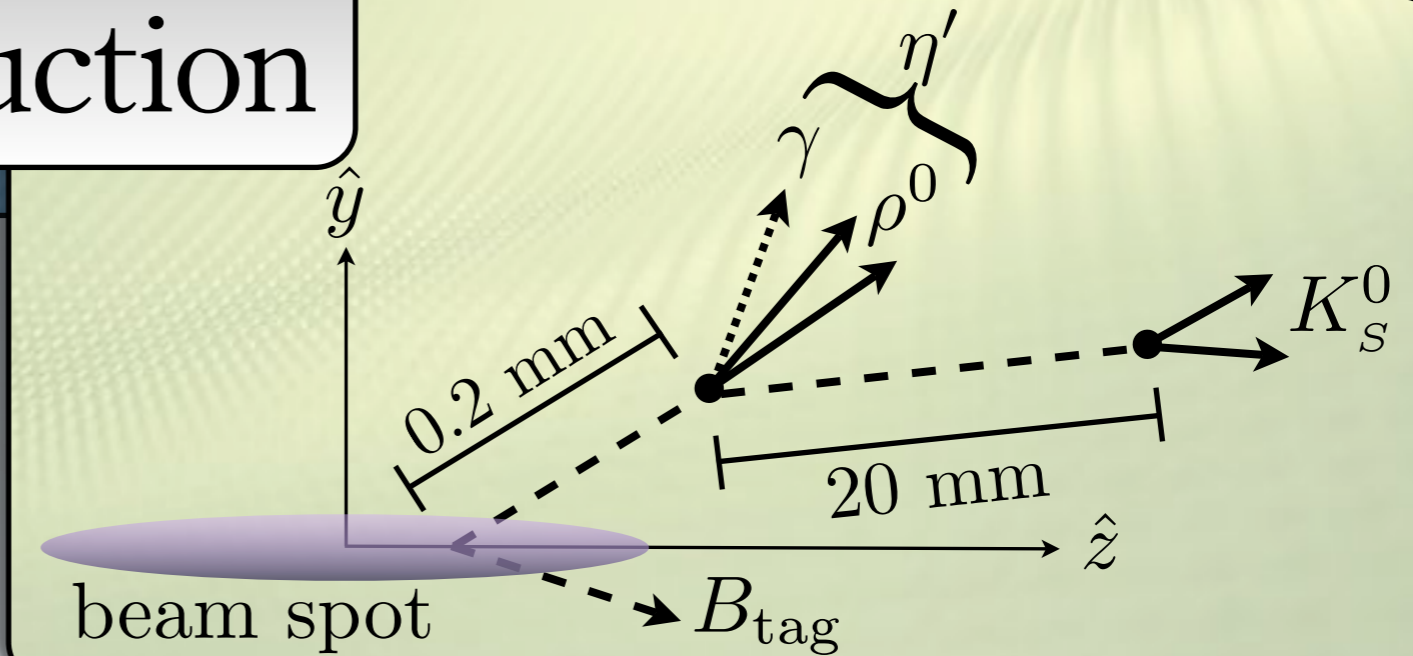
- Fit samples of 500 - 20k events (depending on decay channel).
- Characterize event types (signal, backgrounds) using distributions of variables related to decay kinematics and event-shape.
- Simultaneously
 - isolate signal decays from large background,
 - extract parameters of interest.

3. Potential backgrounds:

- 99% of background is continuum $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$).
- Charmless decays from $B\bar{B}$ events in several sub-modes.
- Charmed B decays in a few sub-modes.

Candidate Reconstruction

- Combine tracks and photons to make B candidates composed of various intermediate resonances.
- In fit to entire decay tree, obtain B decay vertex with resolution of ~ 0.1 mm.
- Constrain η' , η , π^0 masses to nominal values; account for K_S^0 lifetime.



Selection Requirements:

- $N_{\text{trks}} \geq N_{\text{tracks in decay mode}} + 1$,
- $|\Delta E| \leq 0.2$ GeV,
- $5.25 \leq m_{\text{ES}} < 5.2893$ GeV,
- $E_\gamma > 30$ MeV for π^0 ,
- $E_\gamma > 50$ MeV for $\eta_{\gamma\gamma}$,
- $E_\gamma > 100$ MeV in $\eta'_{\rho\gamma}$,
- $|\cos \theta_\rho| < 0.9$, where θ_ρ is the angle between a ρ^0 -daughter π^+ and the η' in the ρ^0 rest frame,
- invariant masses of resonances to be 2-3 widths from nominal mass.
- K_{S+-}^0 vertex fit probability > 0.001 ,
- K_{S+-}^0 flight length at least 3 times the uncertainty on the flight length,
- $|\Delta t| < 20$ ps,
- $\sigma_{\Delta t} < 2.5$ ps, where $\sigma_{\Delta t}$ is the uncertainty on Δt .

Maximum likelihood fit

- For N measurements of quantities \mathbf{x} distributed according to a *probability density function* (PDF) $\mathcal{P}(\mathbf{x}, \boldsymbol{\alpha})$ where $\boldsymbol{\alpha}$ is a set of parameters, the likelihood is defined:

$$\mathcal{L} = \prod_{i=1}^N \mathcal{P}(\mathbf{x}_i, \boldsymbol{\alpha})$$

- Given the data \mathbf{x}_i , the maximum of \mathcal{L} over $\boldsymbol{\alpha}$ gives an unbiased estimate of $\boldsymbol{\alpha}$.
- For m components of the data, each of fraction f_j ($\sum f_j = 1$), the PDF has form:

$$\mathcal{P}(\mathbf{x}_i, \boldsymbol{\alpha}) = \sum_{j=1}^m f_j \mathcal{P}_j(\mathbf{x}_i, \boldsymbol{\alpha}_j)$$

- Since N is randomly distributed, include Poisson factor for making N measurements when expecting $\sum \nu_j$:

$$\mathcal{L} = \frac{e^{-(\sum \nu_j)}}{N!} \prod_{i=1}^N \sum_{j=1}^m \nu_j \mathcal{P}_j(\mathbf{x}_i, \boldsymbol{\alpha}_j)$$

ML fit specifics

- Correlations between variables of \mathbf{x} are low ($<5\%$), so we factorize the PDF for each fit component j :

$$\mathcal{P}_j(\mathbf{x}, \boldsymbol{\alpha}_j) = \mathcal{P}_j(x_1, \boldsymbol{\alpha}_{1,j}) \mathcal{P}_j(x_2, \boldsymbol{\alpha}_{2,j}) \dots \mathcal{P}_j(x_n, \boldsymbol{\alpha}_{n,j})$$

- Minimize $-\ln \mathcal{L}$ instead of maximizing \mathcal{L} .

- The first task of the ML fit is determine n PDFs for m fit components.
- The m fit components are:

1. Signal (sig).

2. Continuum ($q\bar{q}$).

3. Charmless $B\bar{B}$ (chls) for some modes.

4. Charmed $B\bar{B}$ (chrm) for some modes.

- The n observables:

1. Kinematic quantities ($m_{\text{ES}}, \Delta E$).

2. Event-shape Fisher discriminant (\mathcal{F}).

3. Time-difference (Δt).

Kinematic variables: m_{ES} , ΔE

Beams 4-momentum:

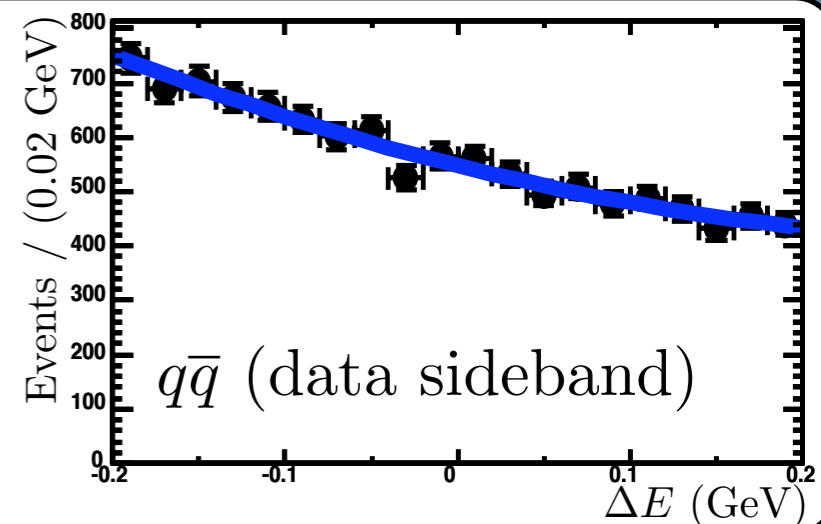
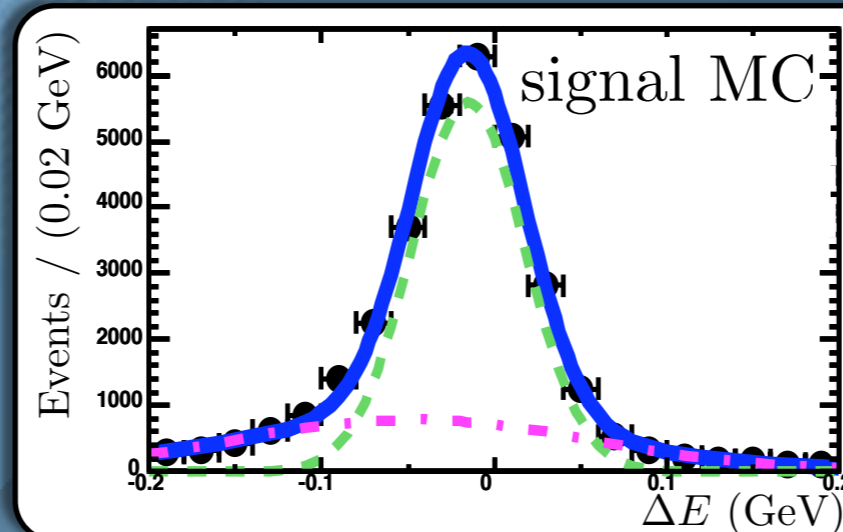
$$q_0 = (E_0, \mathbf{p}_0)$$

Signal B 4-momentum:

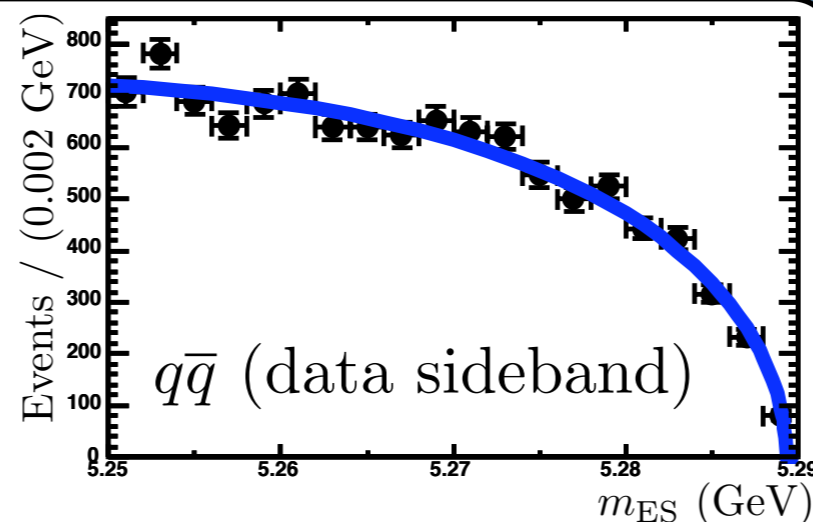
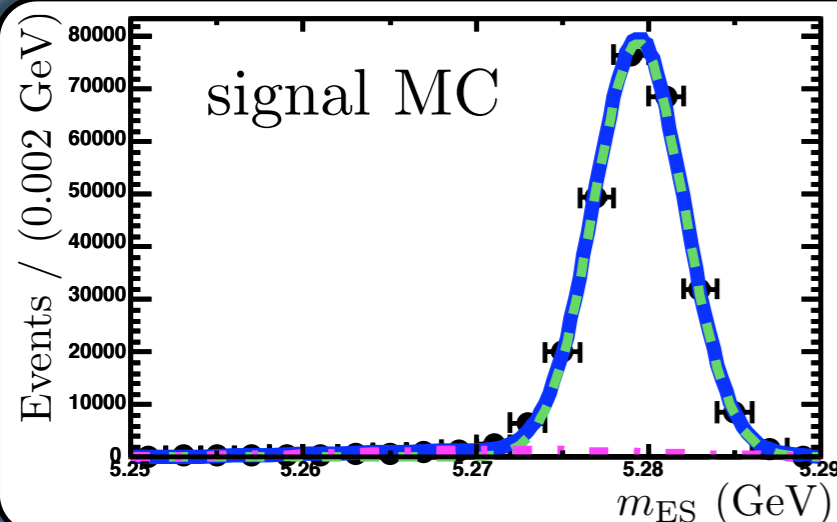
$$q_B = (E_B, \mathbf{p}_B)$$

$$s \equiv q_0^2$$

* Denotes CM frame.



$$\Delta E = E_B^* - \frac{1}{2} E_0^*$$

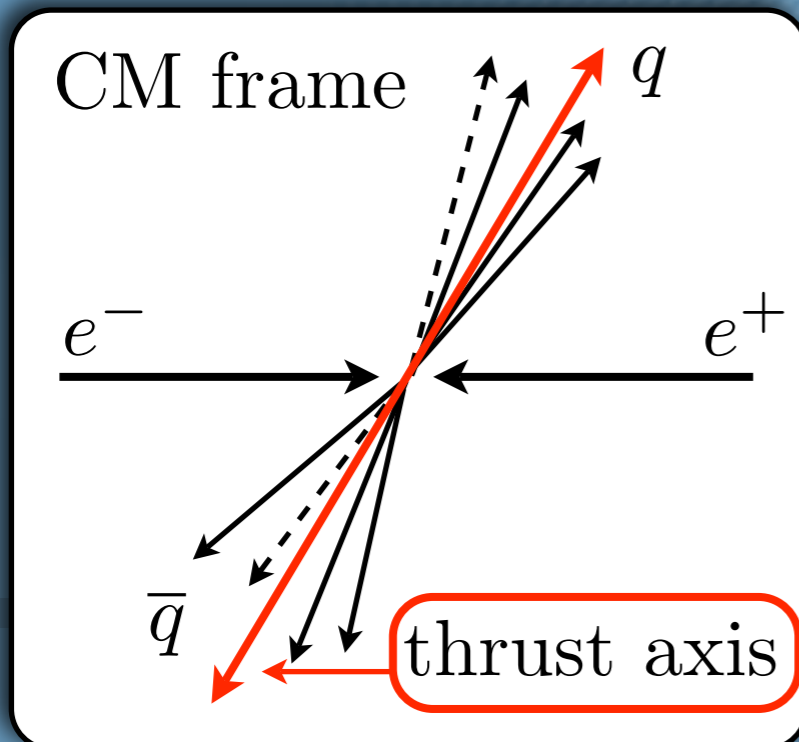


Designed to be uncorrelated.

Lab quantities require no mass hypothesis.

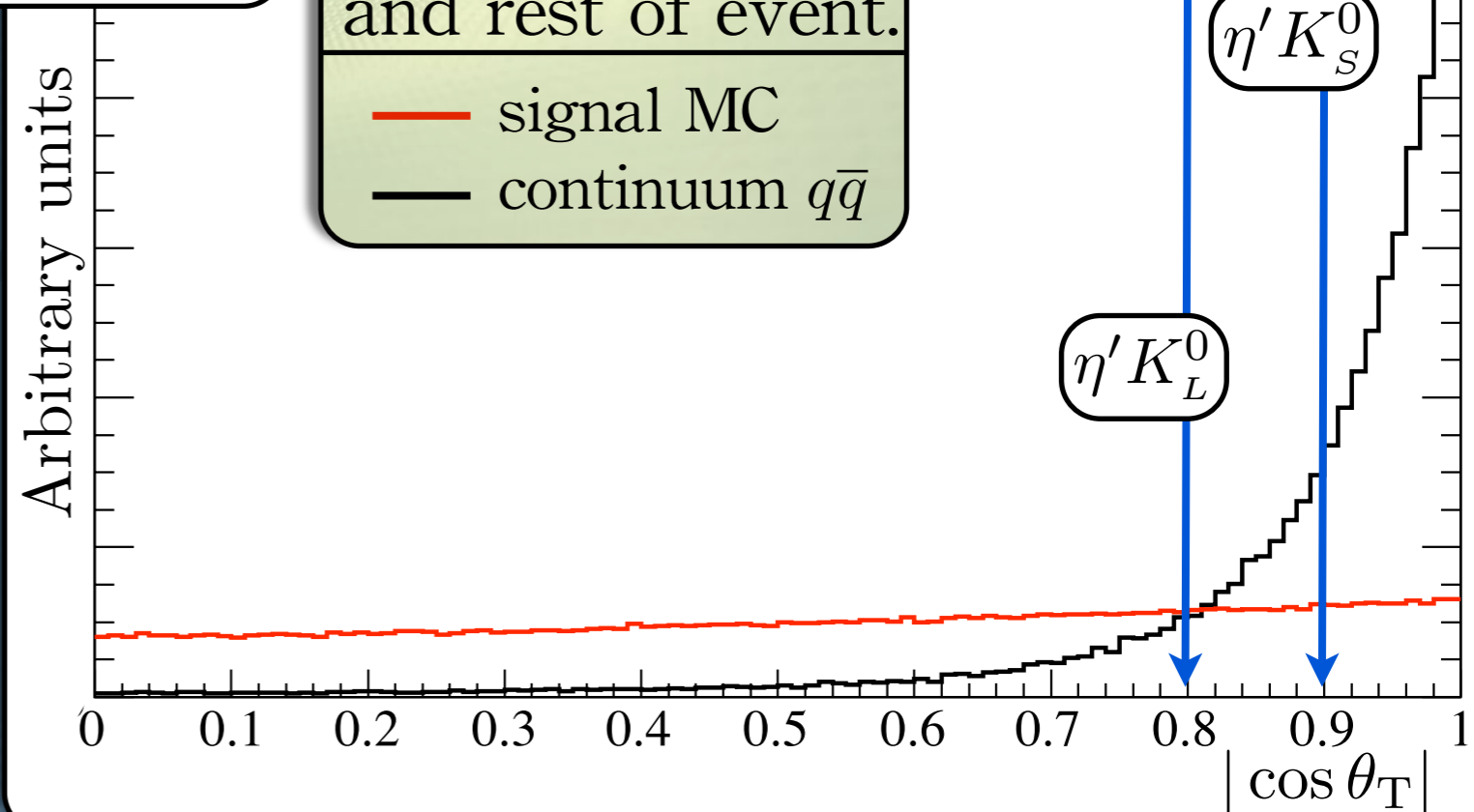
$$m_{\text{ES}} = \sqrt{(\frac{1}{2}s + \mathbf{p}_0 \cdot \mathbf{p}_B)^2 / E_0^2 - p_B^2} = \sqrt{\frac{s}{4} - p_B^{*2}}$$

Event-shape variables

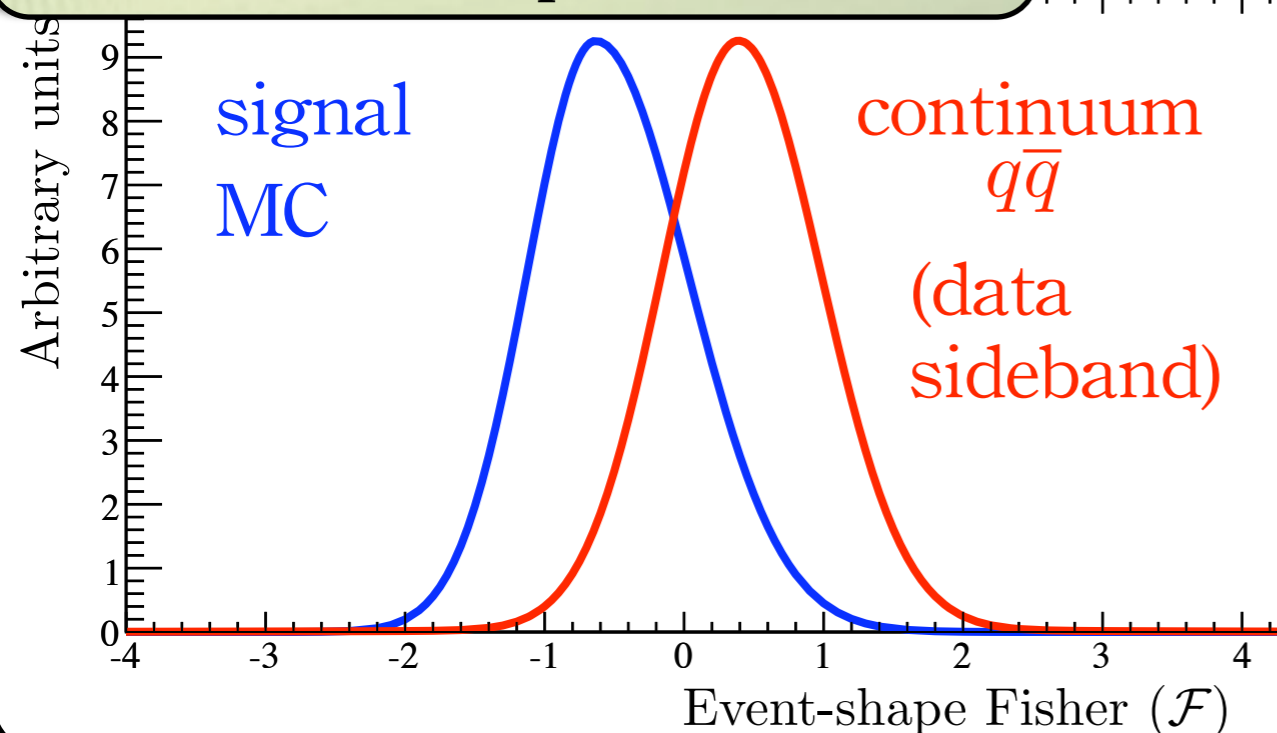


Cut on angle b/w thrust axes of B and rest of event.

— signal MC
— continuum $q\bar{q}$



Fisher discriminant (\mathcal{F}) of other event-shape variables:



1. Angle w.r.t. beams of B momentum.
2. Angle w.r.t. beams of B thrust axis.
3. Zeroth angular momentum L_0 .
4. Second angular momentum L_2 .

$$L_i = \sum_j p_j \times |\cos \theta_j|^i$$

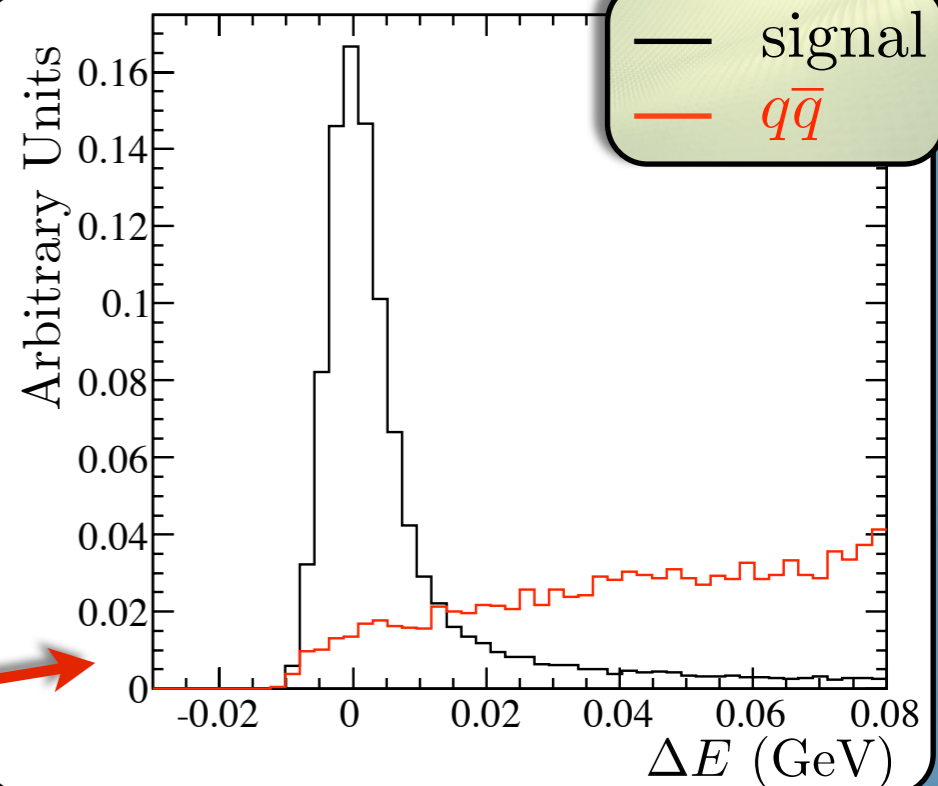
p_j momentum of j^{th} particle in event.

θ_j angle wrt B thrust axis of particle j .

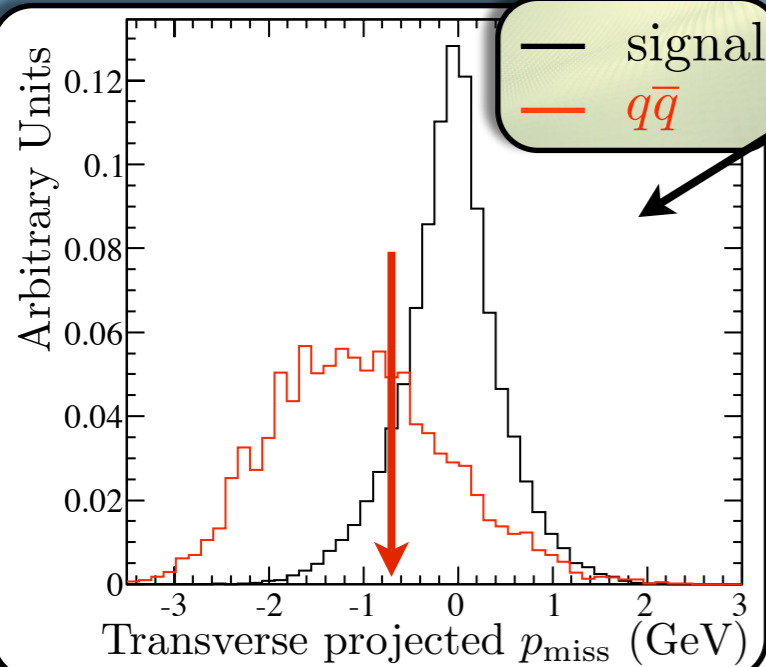
Sum excludes B daughters.

K_L^0 Reconstruction

- K_L^0 's detected in EMC (60%) and IFR (40%).
- Can't reconstruct K_L^0 4-momentum, obtain K_L^0 direction from η' decay vertex and centroid of EMC/IFR cluster.
- Constrain masses of B and K_L^0 in vertexing.
- $m_{ES}/\Delta E$ 100% correlated; only ΔE in ML fit.

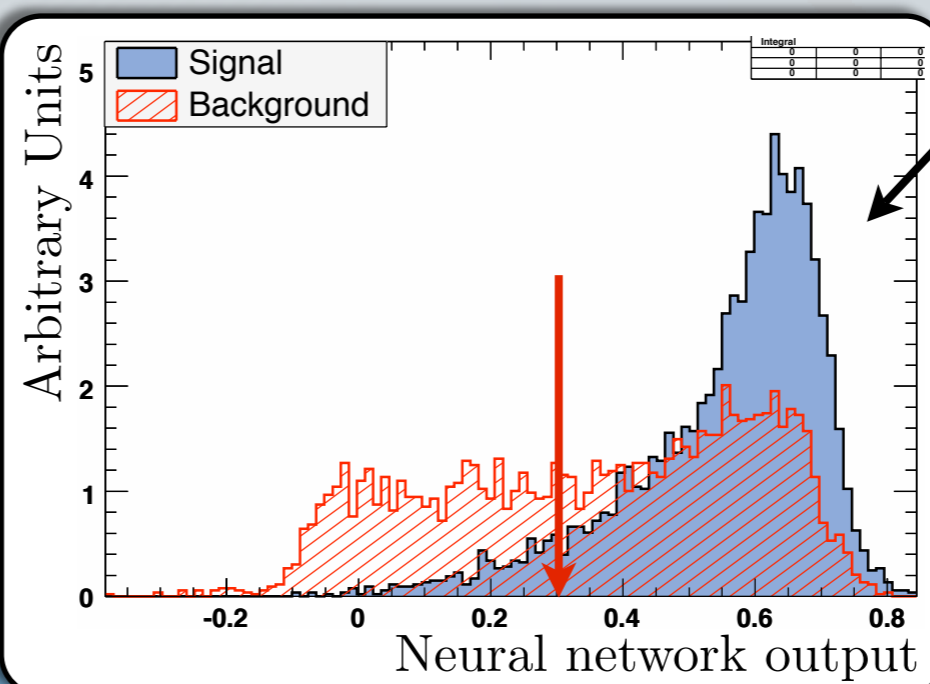
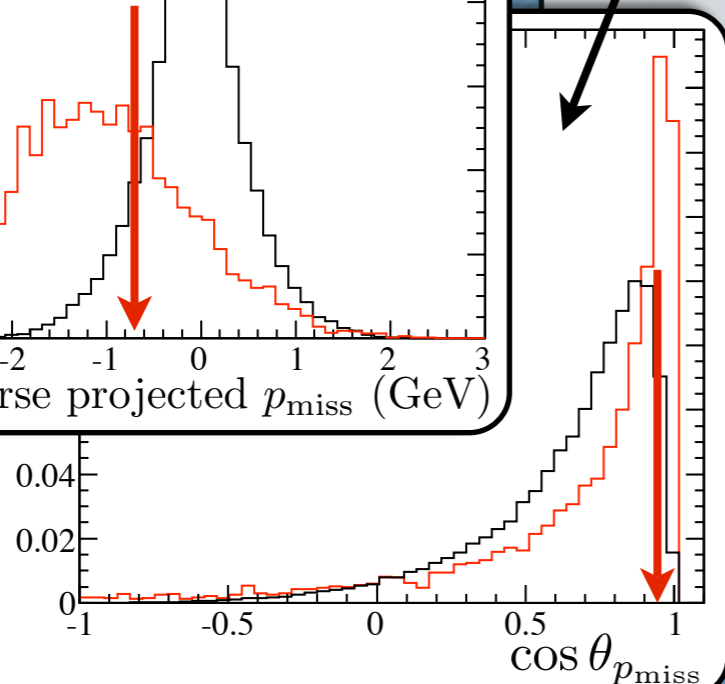


Three cuts to reduce $q\bar{q}$, optimized for precision on S and C:



1. Magnitude of missing momentum p_{miss} .

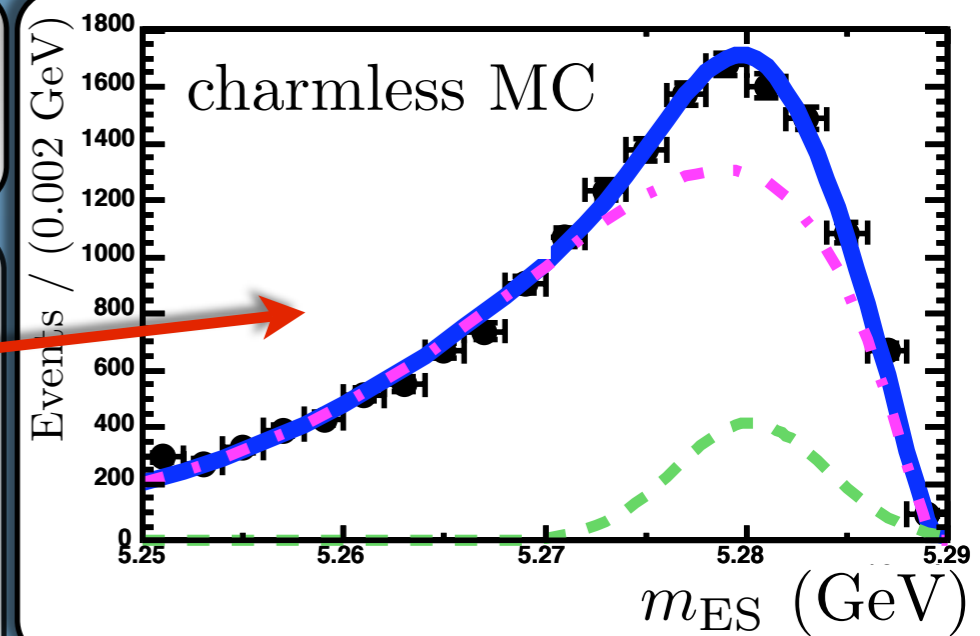
2. Angle wrt beams of missing momentum $\cos \theta_{p_{miss}}$.



3. Output of neural network of EMC shower-shape quantities.

Charmless $B\bar{B}$ background

- Background (BG) events from charmless $B\bar{B}$ decays have broad peaks in ΔE and m_{ES}
- Identify BG modes with generic $B\bar{B}$ MC.
- Use exclusive MC for identified modes and known BF's to model charmless background.
- Example from $B^0 \rightarrow \eta'_{\rho\gamma} K_S^0$:



Expected charmless BG for each sub-decay mode:

Bkg. channel	MC ϵ (%)	Est. \mathcal{B} (10^{-6})	$\prod \mathcal{B}_i$	Norm. # $B\bar{B}$ Bkg.	# in PDF Bkg. file
$B^+ \rightarrow a_1^+(\rho^0\pi^+)K^0$	2.25	$34.9^{+6.7}_{-6.7}$	0.231	83.4	5290
$B^0 \rightarrow \pi^+\pi^-K^0$	1.14	$44.8^{+2.6}_{-2.5}$	0.346	81.2	5145
$B^0 \rightarrow a_1^0(\rho^-\pi^+)K^0$	2.08	15*	0.231	33.1	2100
$B^+ \rightarrow \rho^+K^0$	0.82	$8.0^{+1.5}_{-1.4}$	0.500	15	950
$B^0 \rightarrow \phi_{3\pi}K^0$	6.32	$8.3^{+1.2}_{-1.0}$	0.053	12.8	809
$B^+ \rightarrow a_1^+(\rho^+\pi^0)K^0$	0.46	$34.9^{+6.7}_{-6.7}$	0.115	8.5	537
$B^+ \rightarrow \rho K_0^*(1430)$	0.05	40*	1.000	8.3	529
$B^+ \rightarrow \rho^0 K_{K_S^0\pi^+}^{*+}$	1.6	$3.6^{+1.9}_{-1.8}$	0.231	6.1	387
$B^0 \rightarrow K^+K^-K^0$	0.13	$24.7^{+2.3}_{-2.3}$	0.346	5	316
$B^0 \rightarrow \rho K_0^*(1430)$	0.05	20*	1.000	4.6	292
$B^0 \rightarrow \omega K^0$	0.61	$5.1^{+0.6}_{-0.6}$	0.308	4.4	279
$B^+ \rightarrow \eta'_{\rho\gamma} K_{K_S^0\pi^+}^{*+}$	2.67	$4.9^{+2.1}_{-1.9}$	0.067	4	255
Total				279.4	17463

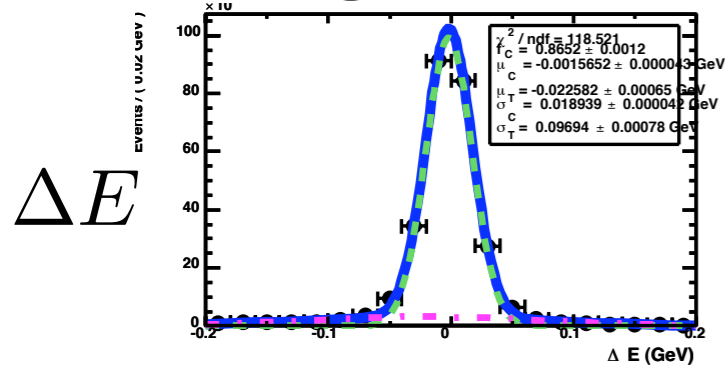
Mode # Events

$\eta'_{\eta\pi\pi} K_S$	< 5
$\eta'_{\rho\gamma} K_S$	279
$\eta'_{\eta\pi\pi} K_{S00}$	< 5
$\eta'_{\rho\gamma} K_{S00}$	69
$\eta'_{5\pi} K_S$	< 5
$\eta'_{\eta\pi\pi} K_L$	22
$\eta'_{5\pi} K_L$	< 5

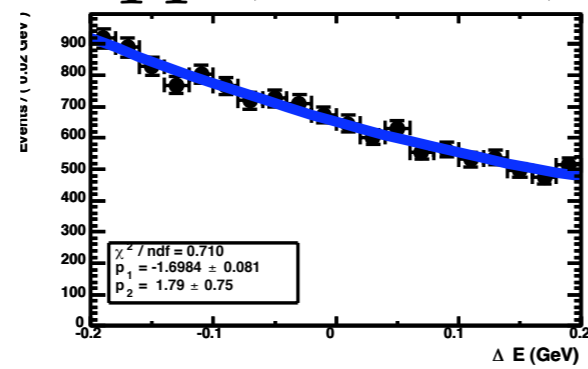
Chls BG entering fit, to be discriminated against ~1000 signal events.

$B^0 \rightarrow \eta'_{\rho\gamma} K_S^0$ PDFs

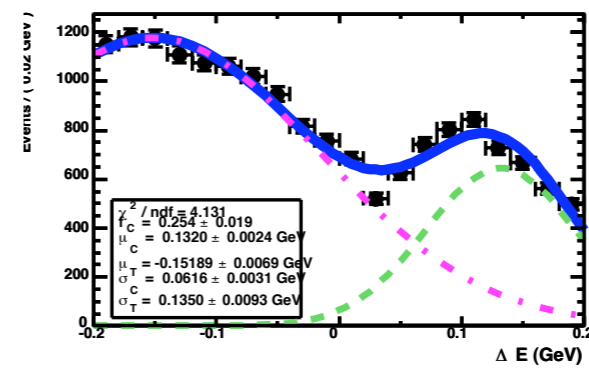
signal MC



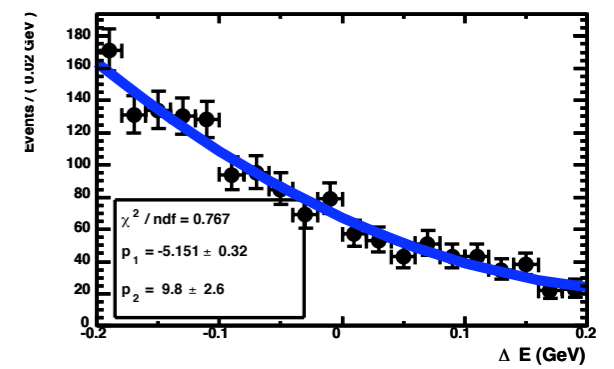
$q\bar{q}$ (sideband)



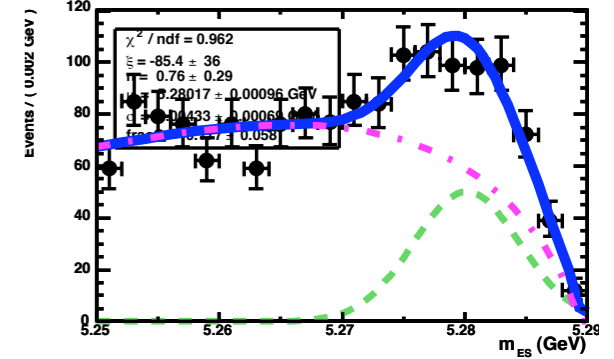
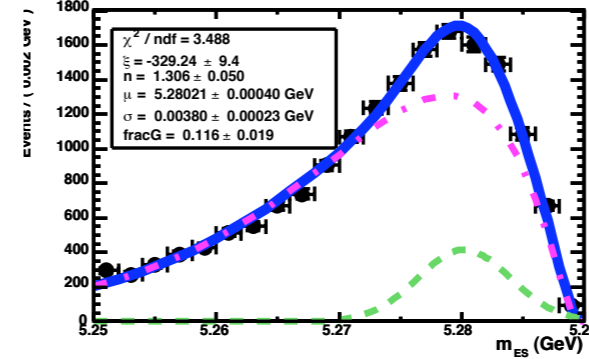
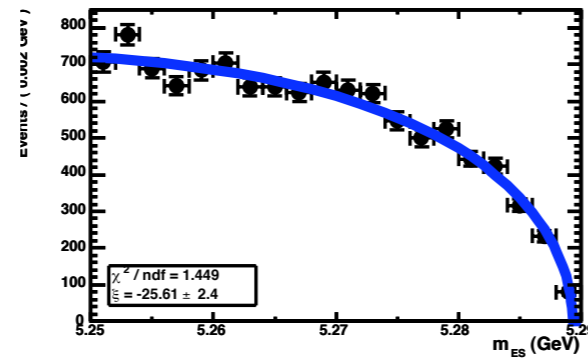
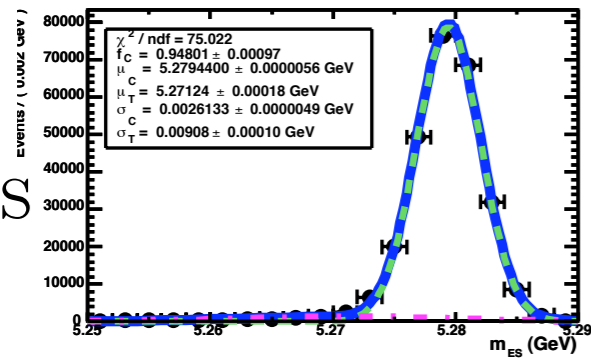
chls $B\bar{B}$ MC



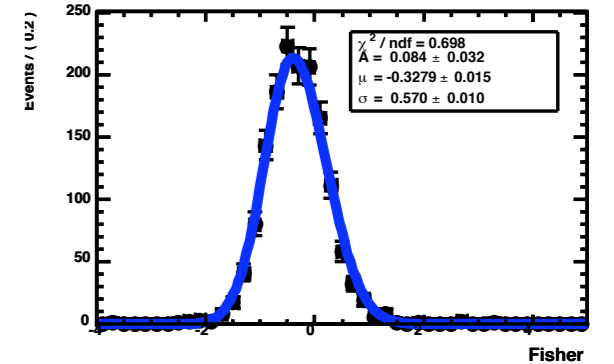
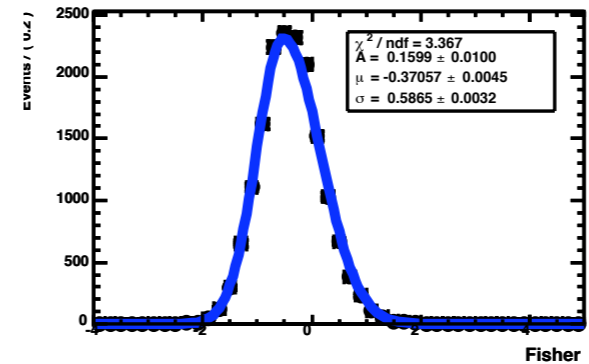
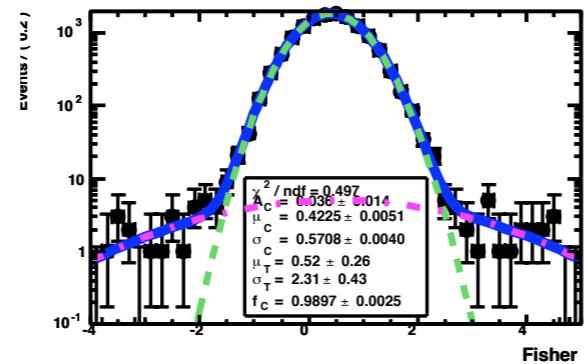
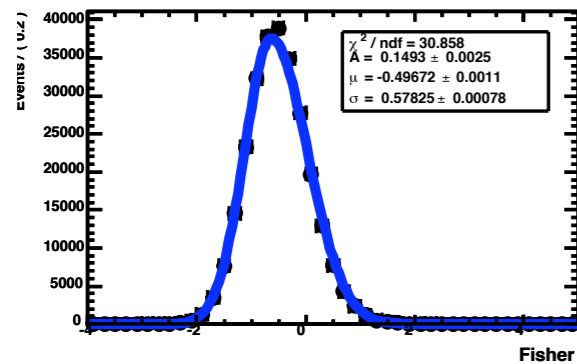
chrm $B\bar{B}$ MC



m_{ES}



\mathcal{F}

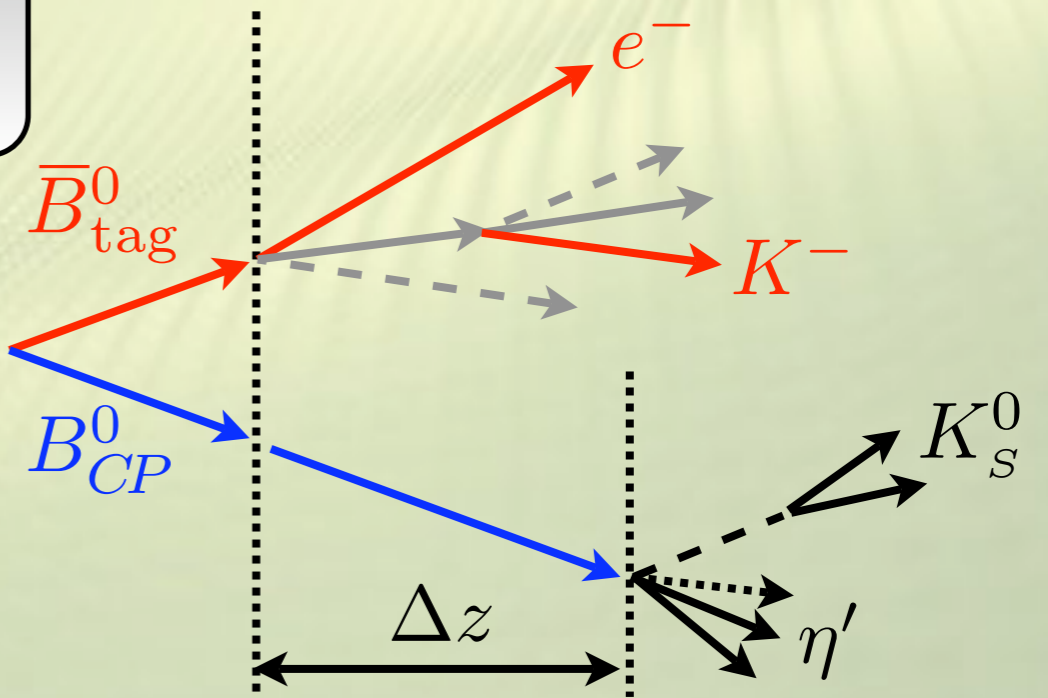


Time-dependent analysis

- Measure Δz , convert to Δt using boost, p^μ of B_{CP} , τ_{B^0} .
- Determine flavor of B_{tag} with tagging algorithm (next slide).
- Δt and tag (\pm) go into:

$$\Delta z \simeq \beta\gamma c\Delta t$$

$$\beta\gamma = 0.56$$



$$f_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} [1 \mp \eta_f S_f \sin(\Delta m_B \Delta t) \mp C_f \cos(\Delta m_B \Delta t)]$$

- Modify f_{\pm} for tagging performance:

$$f'_{\pm} = (1 - w_{\pm})f_{\pm} + w_{\mp}f_{\mp}$$

w_+ (w_-): wrong tag probability for true B^0 (\bar{B}^0)
 $\Delta w \equiv w_+ - w_-$, $w \equiv (w_+ + w_-)/2$

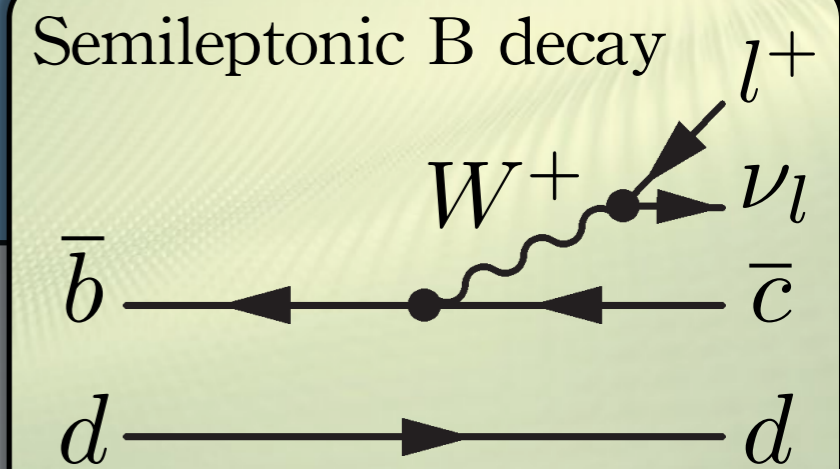
$$f'_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} \left\{ 1 \mp \Delta w \pm (1 - 2w) [-\eta_f S_f \sin(\Delta m_B \Delta t) - C_f \cos(\Delta m_B \Delta t)] \right\}$$

- Modify f'_{\pm} for tagging efficiency asymmetry (μ) for B^0 and \bar{B}^0 .
- Convolve f'_{\pm} with Δt resolution function to obtain final Δt PDF:

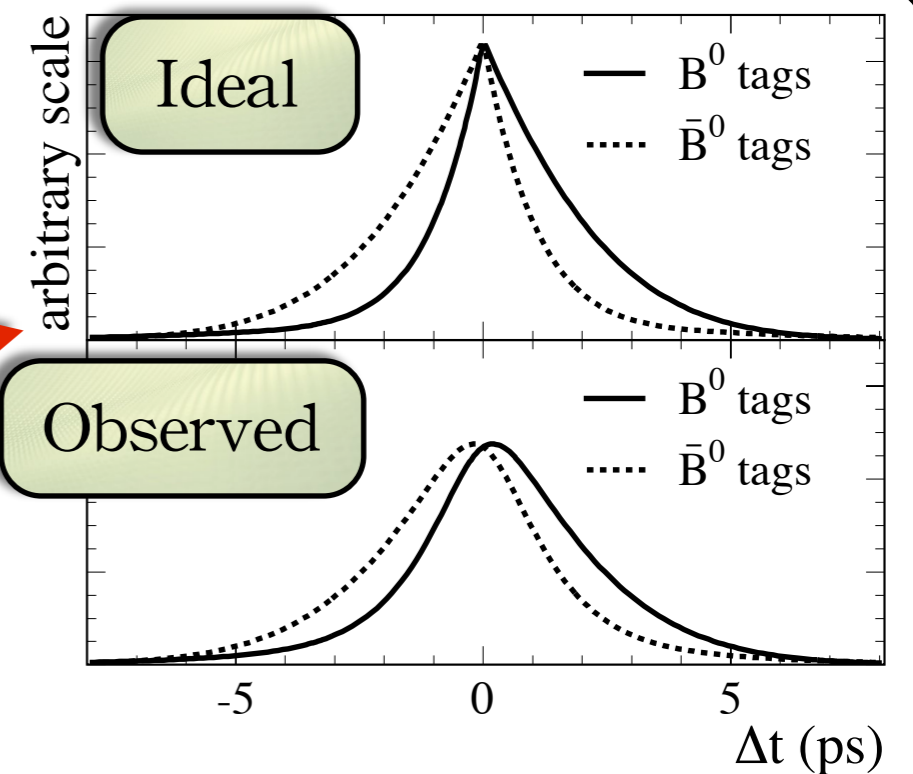
$$\mathcal{P}_{\pm}(\Delta t | \sigma_{\Delta t}) = f'_{\pm}(\Delta t) \otimes \mathcal{R}(\Delta t, \sigma_{\Delta t})$$

Tagging algorithm

- Neural-network-based algorithm assigns each B_{tag} candidate to 1 of 6 categories.
- Category is determined by continuous output of algorithm (and lepton in the B_{tag} final state).
- Cleanest tagging from semileptonic decays, such as $B^0 \rightarrow D^{*-} l^+ \nu_l$.
- Clean tagging from $b \rightarrow c \rightarrow s$ decays, such as $B^0 \rightarrow D^{*-} \rho^+$, $D^{*-} \rightarrow \bar{D}^0 \pi^-$, $\bar{D}^0 \rightarrow K^+ \pi^-$.



- Mistagging and experimental Δt resolution change ideal into observed Δt distribution.



B_{flav} Sample

- How do we get parameters for tagging (ϵ , w , Δw , μ) and $\mathcal{R}(\Delta t, \sigma_{\Delta t})$?
 - The performance of the tagging algorithm is independent of signal mode.
 - Resolution on Δt is dominated by tag-side vertex.
- B_{flav} sample : 100k (c.f. 2.5k signal events) fully reconstructed, self-tagging decays $B^0 \rightarrow D^{(*)-}(\pi^+, \rho^+, a_1^+)$.
- Fit B_{flav} sample to determine tagging and resolution model parameters.
 - No dependence on MC.

ϵ : probability to be tagged

$$\mu \equiv \epsilon(B^0) - \epsilon(\bar{B}^0)$$

$$Q \equiv \epsilon(1 - 2w)^2$$

$$\sigma_{S,C} \propto 1/\sqrt{Q}$$

Category	ϵ (%)	w (%)	Δw (%)	μ (%)	Q (%)
Lepton	9.0 ± 0.1	2.8 ± 0.3	0.3 ± 0.5	-0.3 ± 0.9	8.0 ± 0.1
Kaon I	10.8 ± 0.1	5.3 ± 0.3	-0.1 ± 0.6	0.1 ± 0.9	8.7 ± 0.1
Kaon II	17.2 ± 0.1	14.5 ± 0.3	0.4 ± 0.6	0.6 ± 0.8	8.7 ± 0.2
Kaon-Pion	13.7 ± 0.1	23.3 ± 0.4	-0.7 ± 0.7	0.2 ± 0.9	3.9 ± 0.1
Pion	14.2 ± 0.1	32.5 ± 0.4	5.1 ± 0.7	-2.5 ± 0.9	1.7 ± 0.1
Other	9.5 ± 0.1	41.5 ± 0.5	3.8 ± 0.8	1.8 ± 1.0	0.3 ± 0.0
All	74.4 ± 0.1				31.2 ± 0.3

75% of events are tagged.

30% effective rate due to mistagging.

Grand Simultaneous Fit

c : tagging category

N_c : events in c

ϵ_c : tag efficiency for c

- For each sub-mode f , \mathcal{L}_f is product of $\mathcal{L}_{c,f}$ for each tagging category:

$$\mathcal{L}_f = \prod_{c=0}^6 \frac{e^{(-\sum_j \nu_{j,c})}}{N_c!} \prod_i^{N_c} (\nu_{\text{sig}} \epsilon_c \mathcal{P}_{\text{sig},c}^i + \nu_{q\bar{q},c} \mathcal{P}_{q\bar{q}}^i + \nu_{\text{chls}} \epsilon_c \mathcal{P}_{\text{chls},c}^i + \nu_{\text{chrn}} \epsilon_c \mathcal{P}_{\text{chrn},c}^i).$$

We average sub-mode results by maximizing product of \mathcal{L}_f 's while constraining all $-\eta_f S_f$ to a single value, $S_{\eta' K^0}$.

133 free parameters !!

non- Δt PDFs:

- Event yields for sig, $q\bar{q}$, chrn, are free.
- Parameters of $q\bar{q}$ are free in fit.
- sig, chrn, chls parameters fixed to values from MC.

Mode	Components
$\eta'_{\eta\pi\pi} K_S$	sig, $q\bar{q}$
$\eta'_{\rho\gamma} K_S$	sig, $q\bar{q}$, chls*, chrn
$\eta'_{\eta\pi\pi} K_{S00}$	sig, $q\bar{q}$
$\eta'_{\rho\gamma} K_{S00}$	sig, $q\bar{q}$, chls*, chrn
$\eta'_{5\pi} K_S$	sig, $q\bar{q}$
$\eta'_{\eta\pi\pi} K_L$	sig, $q\bar{q}$, chls*
$\eta'_{5\pi} K_L$	sig, $q\bar{q}$ *fixed yield

Δt models

Component	Parameter Source		
	Tagging	$\mathcal{R}(\Delta t)$	S, C
sig	B_{flav}	B_{flav}	free
$q\bar{q}$	τ_B fixed to 0	free	n/a
chls	B_{flav}	chls MC	fixed to 0
chrn	B_{flav}	chrn MC	fixed to 0

Fit validation : toy MC experiments

- Perform 150-550 simulated experiments using toy datasets:
 - Generate $q\bar{q}$ and charmed $B\bar{B}$ events from factorized PDFs,
 - Embed MC events for signal and charmless $B\bar{B}$.
- Estimate bias due to correlations in signal and residual contributions of backgrounds and misreconstructed signal.
- Confirm that uncertainties reported by fit are reasonable.

Final state	# toy experiments	# sig input	# chls input	Signal Bias	Bias on S	Bias on C
$\eta'_{\eta\pi\pi}K_S^0$	550	470	0	-3.7 ± 0.5	0.010 ± 0.007	-0.012 ± 0.005
$\eta'_{\rho\gamma}K_S^0$	260	970	279	35.1 ± 1.4	0.002 ± 0.007	-0.007 ± 0.006
$\eta'_{\eta\pi\pi}K_{S00}^0$	200	108	0	-3.0 ± 0.6	0.080 ± 0.034	0.022 ± 0.020
$\eta'_{\rho\gamma}K_{S00}^0$	270	199	69	7.3 ± 1.3	0.054 ± 0.022	-0.007 ± 0.016
$\eta'_{5\pi}K_S^0$	190	173	0	-1.8 ± 0.4	0.021 ± 0.020	-0.009 ± 0.014
$\eta'_{\eta\pi\pi}K_L^0$	235	353	22	-25.3 ± 1.7	-0.019 ± 0.018	-0.007 ± 0.013
$\eta'_{5\pi}K_L^0$	140	170	0	-9.5 ± 1.5	0.099 ± 0.035	0.007 ± 0.022
Weighted Avg.					0.013 ± 0.004	-0.007 ± 0.003
Simultaneous $\eta'K^0$ Fit	175			unbiased fit	0.006 ± 0.006	-0.008 ± 0.006
Simultaneous $\eta'K_S^0$ Fit	175				0.008 ± 0.006	-0.007 ± 0.005
Simultaneous $\eta'K_L^0$ Fit	175				0.003 ± 0.014	-0.006 ± 0.011

Systematic Errors

PDF parameterization:

- Vary fixed parameters by amounts found in studies of data control samples (B_{flav} , $B^- \rightarrow D^0 \pi^-$, $B^+ \rightarrow \eta' K^+$).

BB background:

- Vary S and C for chls, chrm components.
- Vary fixed charmless yields by $\pm 20\%$.

Signal Δt parameterization :

- Comparing toy MC studies performed with Δt parameters ($\mathcal{R}(\Delta t)$ and tagging) from signal and B_{flav} MC.
- Estimate effects due to differences between signal and B_{flav} data.

Source of error	$\sigma(S)$	$\sigma(C)$
Beam position/size	0.002	0.001
SVT alignment	+0.002 -0.001	+0.003 -0.002
Tag-side interference	0.001	0.015
Self-crossfeed	0.006	0.003
Fit Bias	0.006	0.006
PDF Shapes	0.005	0.009
$B\bar{B}$ Background	0.008	0.004
Signal Δt Shape	0.009	0.010
Total	0.016	0.022

$B^0 \rightarrow \eta' K^0$ Results

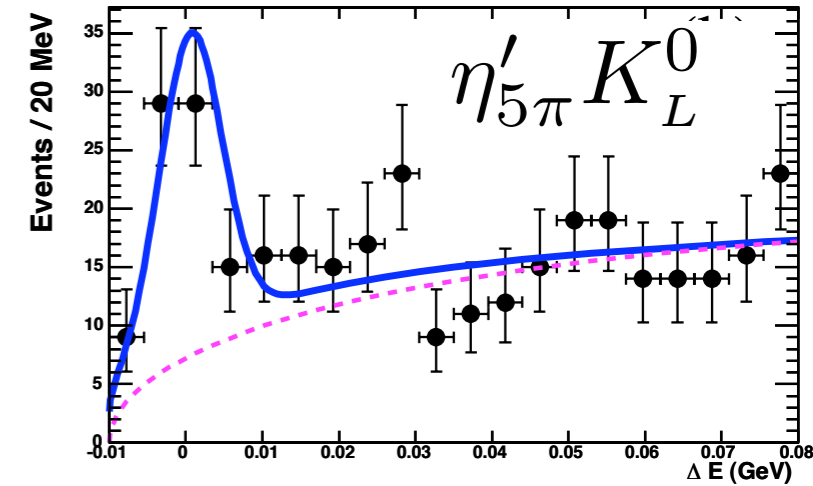
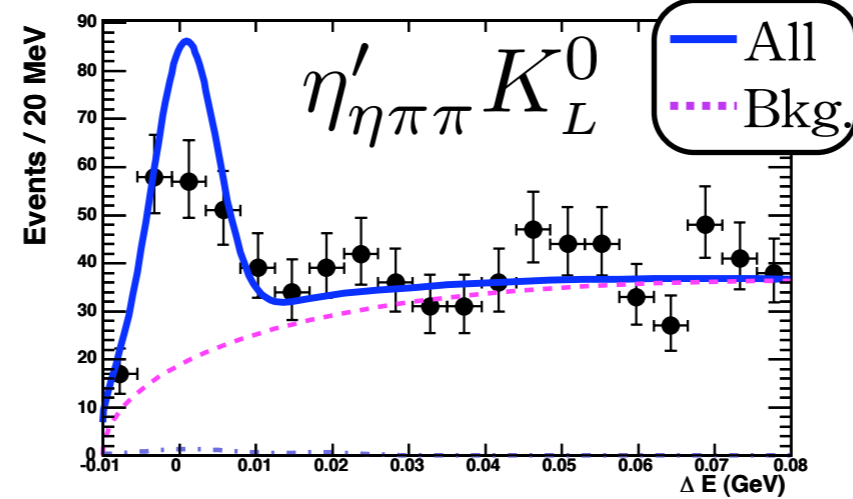
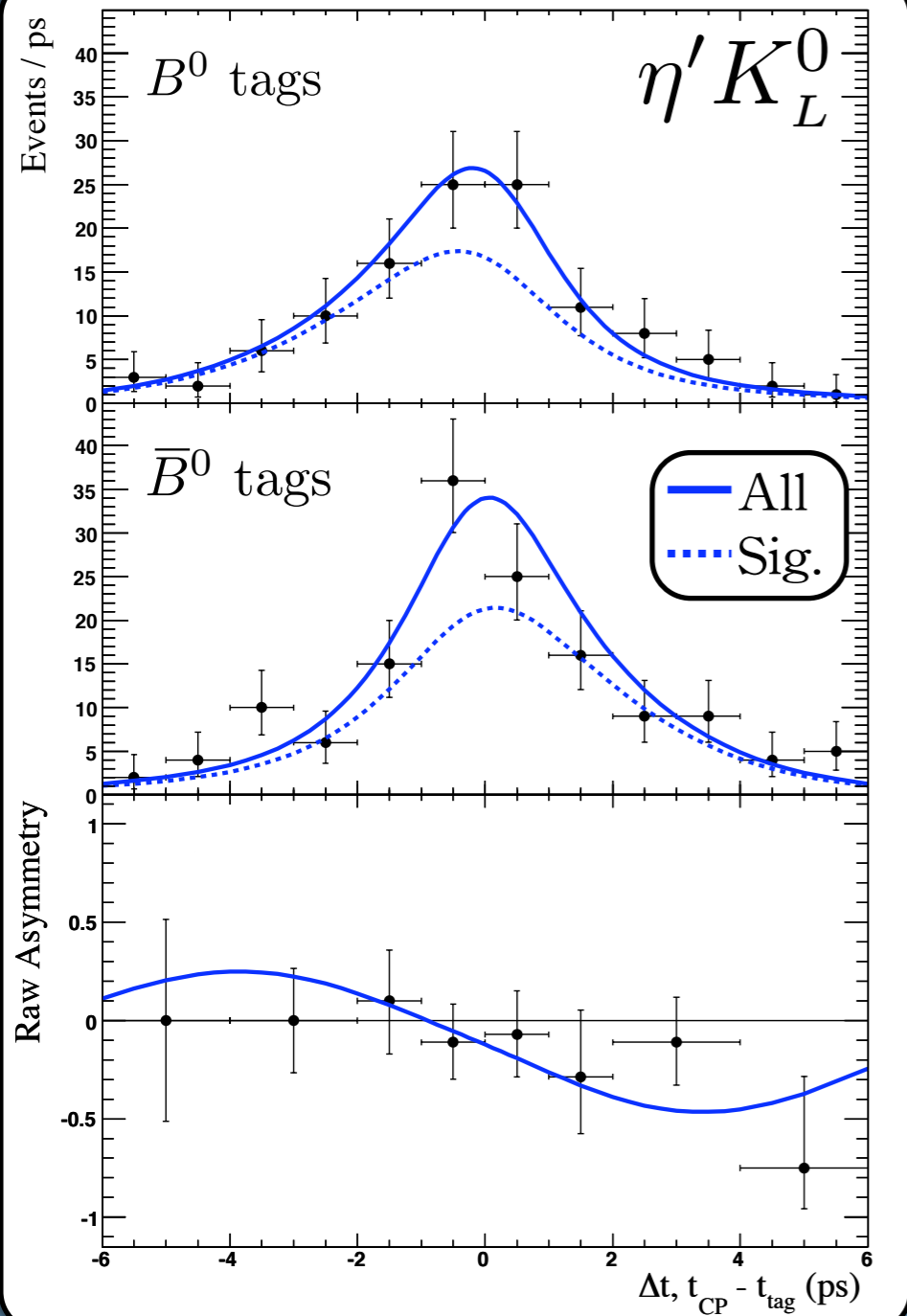
arXiv:0809.1174 (submitted to PRD)

	$\eta'_{\eta\pi\pi} K_S^0$	$\eta'_{\rho\gamma} K_S^0$	$\eta'_{\eta\pi\pi} K_{S00}^0$	$\eta'_{\rho\gamma} K_{S00}^0$	$\eta'_{5\pi} K_S^0$	$\eta'_{\eta\pi\pi} K_L^0$	$\eta'_{5\pi} K_L^0$
Events to fit	1470	22775	1056	27057	513	12217	4586
Signal yield	472 ± 24	1005 ± 40	105 ± 13	206 ± 28	171 ± 14	341 ± 32	158.7 ± 21.6
Chls yield	–	279	–	69	–	22	–
Chrm yield	–	253 ± 67	–	530 ± 84	–	–	–
$-\eta_f S_f$	0.70 ± 0.17	0.46 ± 0.12	0.51 ± 0.34	0.26 ± 0.33	0.76 ± 0.26	0.65 ± 0.22	0.66 ± 0.46
C_f	-0.17 ± 0.11	-0.13 ± 0.09	-0.19 ± 0.30	0.04 ± 0.26	0.05 ± 0.20	0.07 ± 0.19	0.02 ± 0.26
Combined:	<div>$S_{\eta' K_S^0} = 0.537 \pm 0.084$ $C_{\eta' K_S^0} = -0.118 \pm 0.062$</div>					<div>$-1 \cdot S_{\eta' K_L^0} = 0.642 \pm 0.198$ $C_{\eta' K_L^0} = 0.047 \pm 0.154$</div>	
	<div>$S_{\eta' K^0} = 0.551 \pm 0.777$ $C_{\eta' K^0} = -0.094 \pm 0.058$</div>						

- With about 1500 $B^0 \rightarrow \eta' K_S^0$ and 300 $B^0 \rightarrow \eta' K_L^0$ flavor-tagged decays, we find these bias-corrected results:

$$\begin{aligned}
 S_{\eta' K^0} &= 0.545 \pm 0.077 \pm 0.016 \\
 C_{\eta' K^0} &= -0.086 \pm 0.058 \pm 0.022 \\
 S_{\eta' K_S^0} &= 0.529 \pm 0.084 \pm 0.016 \\
 C_{\eta' K_S^0} &= -0.111 \pm 0.062 \pm 0.024 \\
 S_{\eta' K_L^0} &= 0.639 \pm 0.198 \pm 0.033 \\
 C_{\eta' K_L^0} &= 0.053 \pm 0.154 \pm 0.029
 \end{aligned}$$

$B^0 \rightarrow \eta' K_L^0$ Results



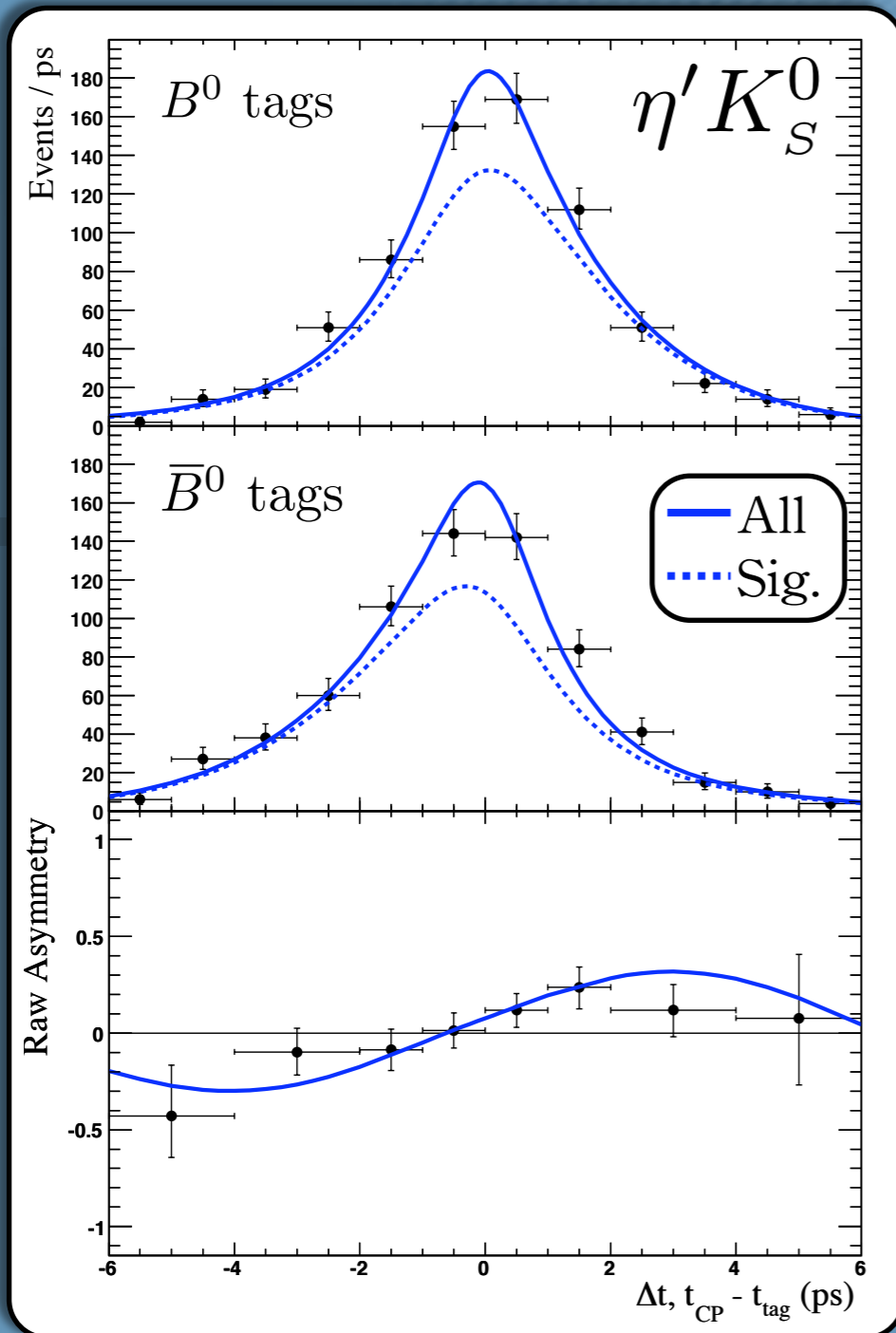
Signal-enhanced projections

Projections of ΔE and Δt with optimized, mode-dependent requirement on

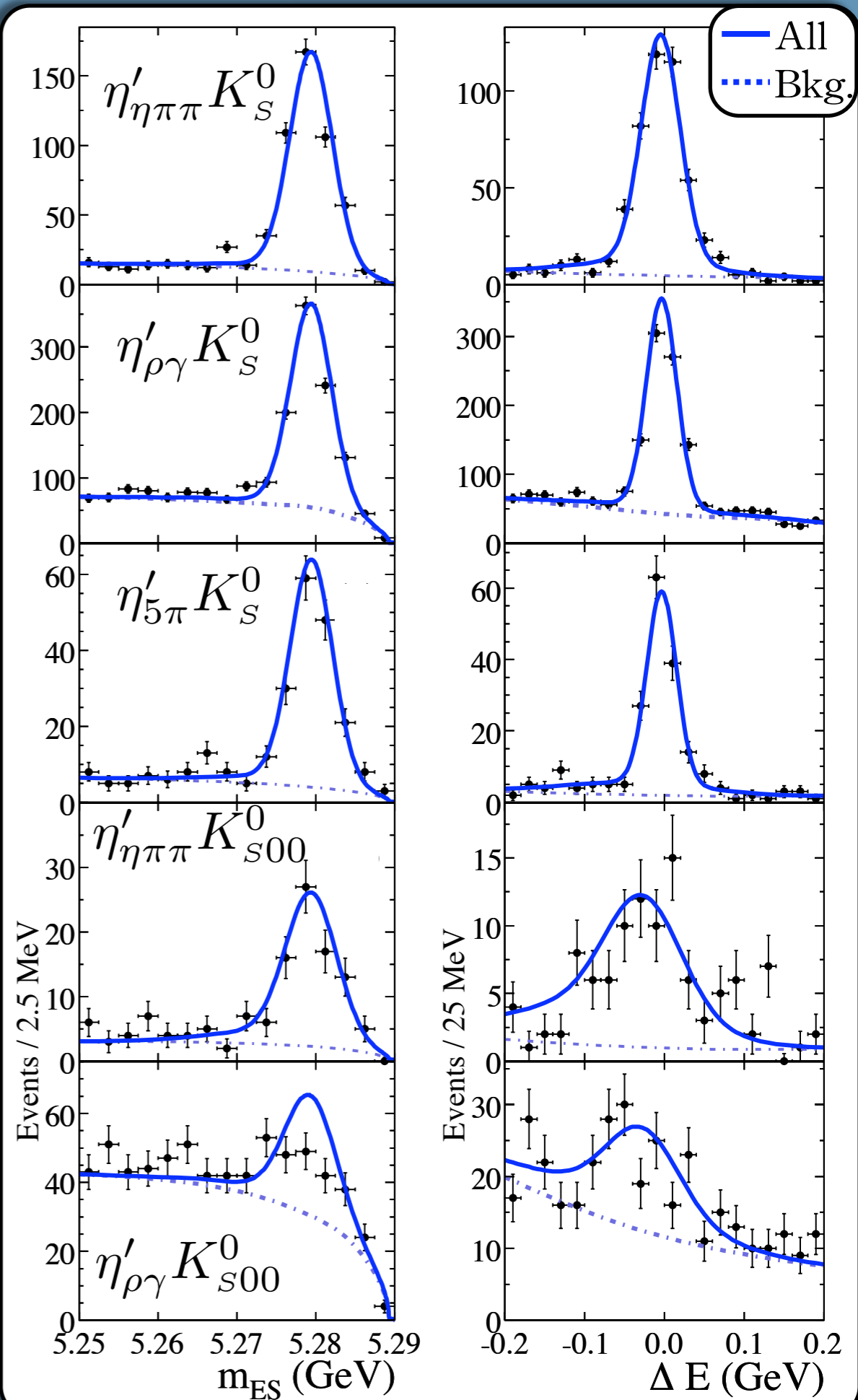
$$\frac{\mathcal{L}_S}{\mathcal{L}_S + \mathcal{L}_B}$$

to enhance the signal.

$B^0 \rightarrow \eta' K_S^0$ Results



Signal-enhanced projections of m_{ES} , ΔE , and Δt



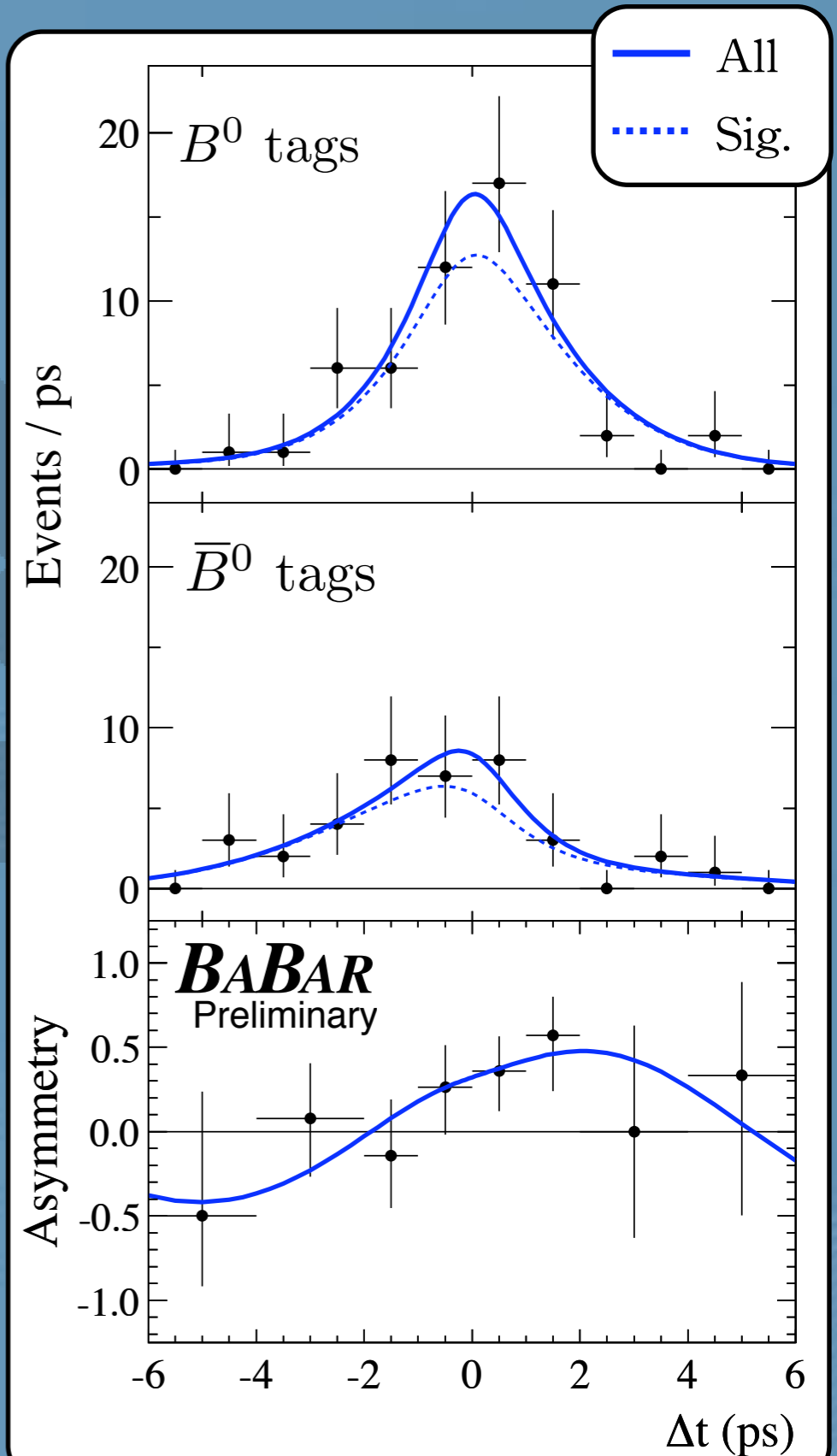
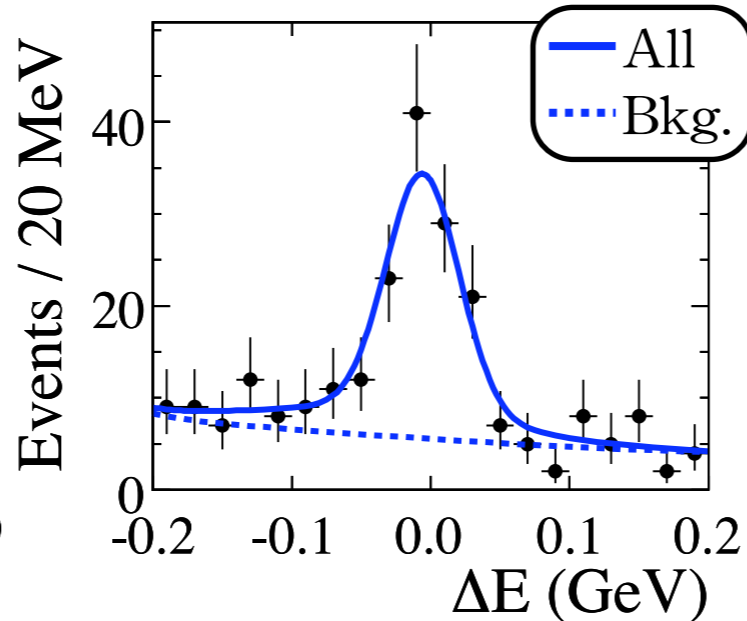
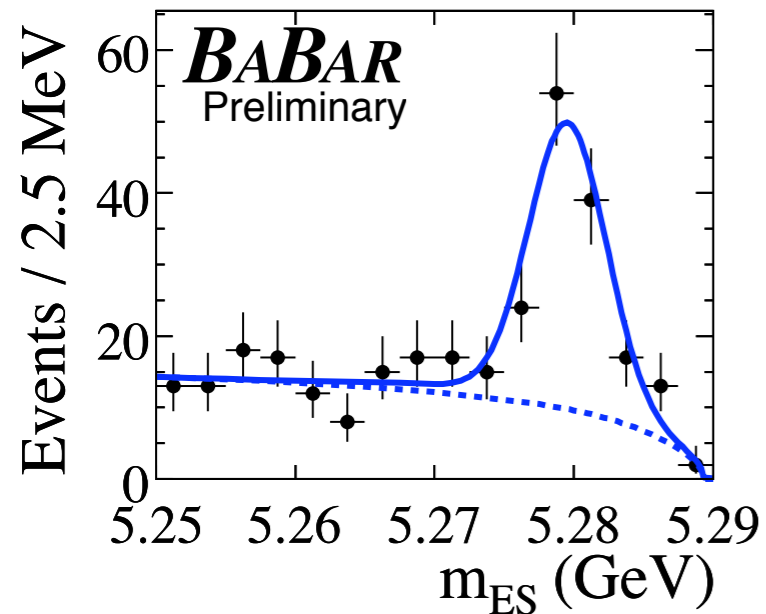
$$B^0 \rightarrow \omega K_S^0$$

arXiv:0809.1174 (submitted to PRD)

- 163 ± 18 signal events with
 $\omega \rightarrow \pi^+ \pi^- \pi^0$ and $K_S^0 \rightarrow \pi^+ \pi^-$

$$S = 0.55^{+0.26}_{-0.29} \pm 0.02$$

$$C = -0.52^{+0.22}_{-0.20} \pm 0.03$$



$$B^0 \rightarrow K^+ K^- K_S^0$$

arXiv:0808.0700

- Reconstruct $K_S^0 \rightarrow \pi^+ \pi^-$ and $\pi^0 \pi^0$.
- Time-dependent amplitude analysis, model uses $\phi K_S^0, f_0 K_S^0, X_0 K_S^0, NR, \chi_{c0} K_S^0, D^+ K^-, D_s^+ K^-$
- Fit entire Dalitz plot, then fit low-mass ($m_{K^+ K^-} < 1.1$ GeV) and high-mass ($m_{K^+ K^-} > 1.1$ GeV) regions.

Fit	Signal yield
Whole DP	1428 ± 47
High-Mass	1011 ± 39
Low-mass	421 ± 25

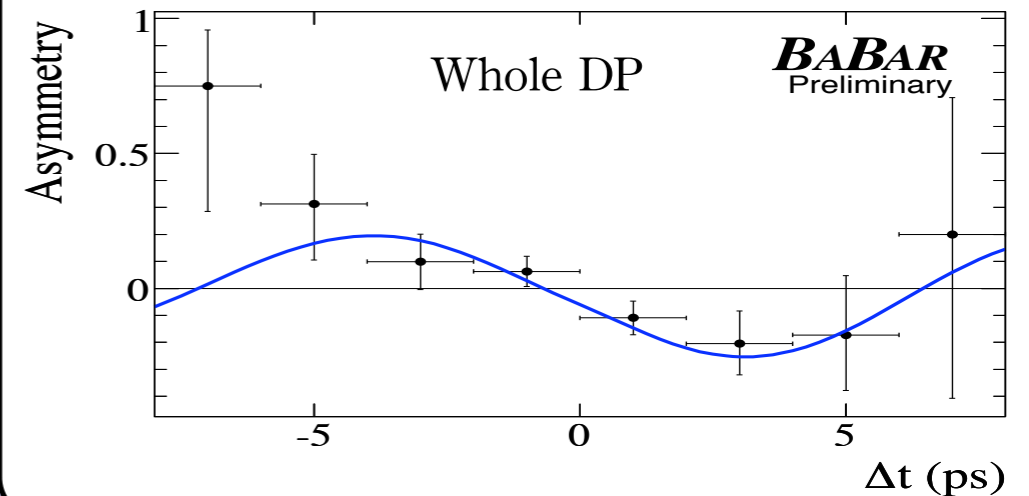
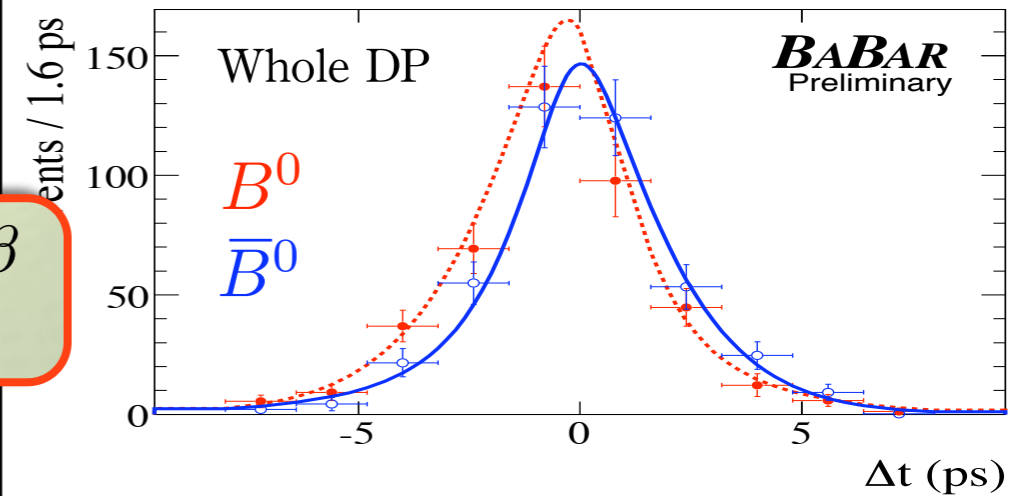
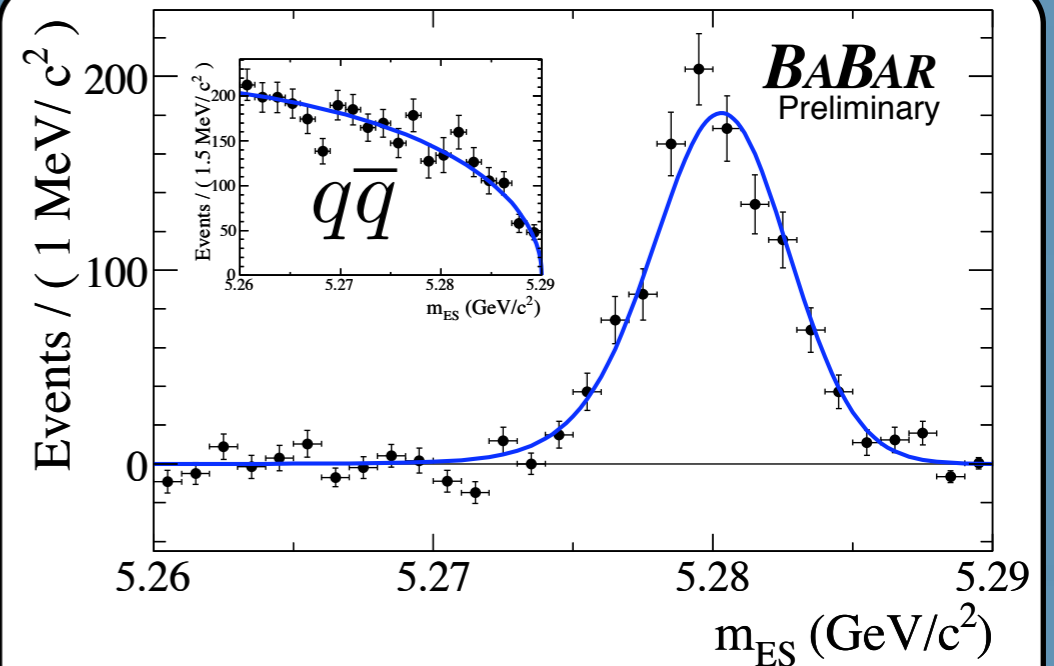
$\frac{\pi}{2} - \beta$ ambiguity in $\sin 2\beta$
ruled out at 4.8σ .

Fit	$A_{CP} (-C_f)$	$\beta_{\text{eff}} (\beta_{\text{SM}} \simeq 0.37)$
Whole DP	$0.03 \pm 0.07 \pm 0.02$	$0.44 \pm 0.07 \pm 0.02$
High-Mass	$0.05 \pm 0.09 \pm 0.04$	$0.52 \pm 0.08 \pm 0.03$
ϕK_S^0	$0.14 \pm 0.19 \pm 0.02$	$0.13 \pm 0.13 \pm 0.02$
$f_0 K_S^0$	$0.01 \pm 0.26 \pm 0.07$	$0.15 \pm 0.13 \pm 0.03$

s Plots¹

Accumulate probability to be signal (computed without plotted variable) in bins of variable of interest. Overlay normalized PDF.

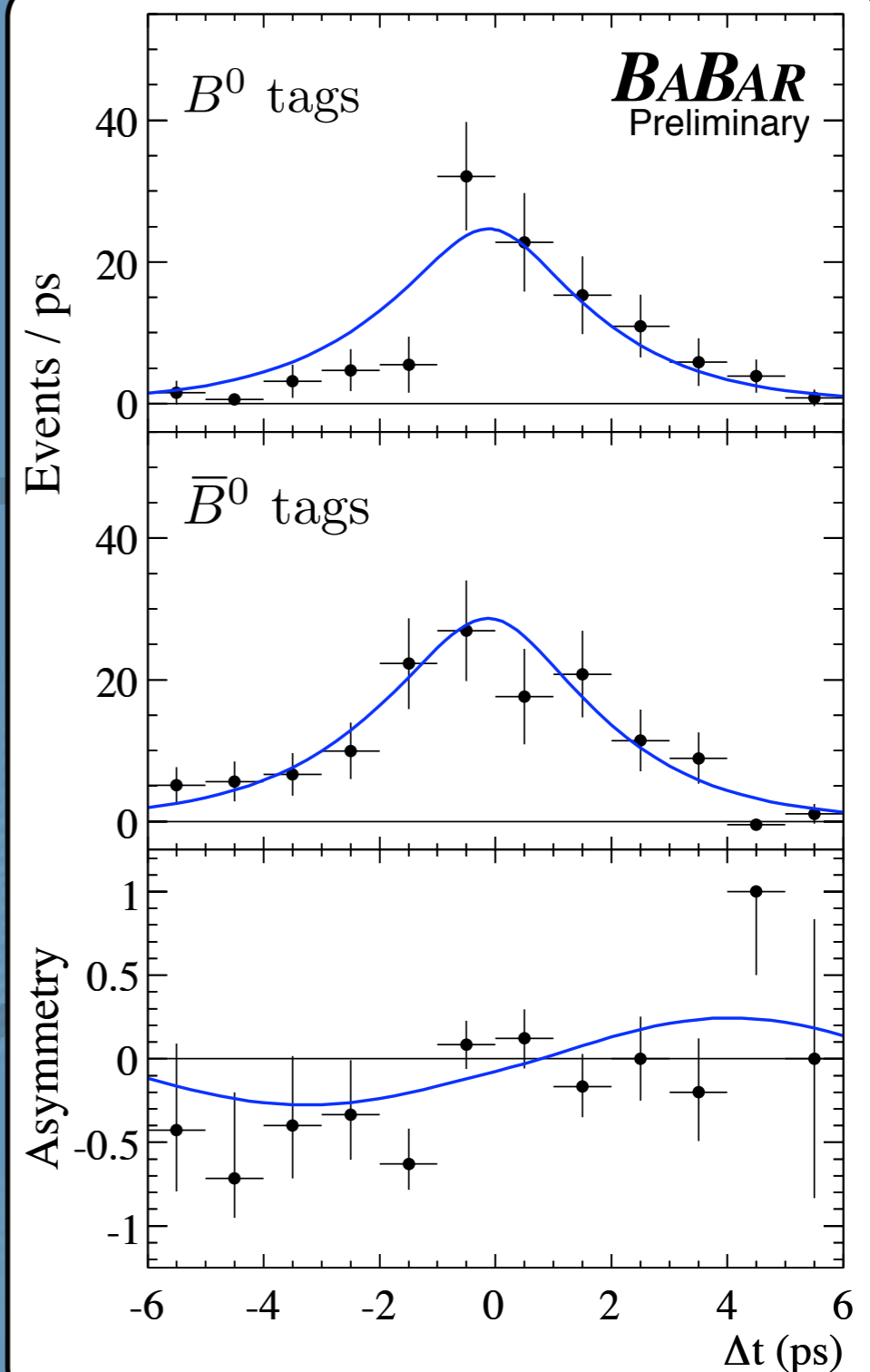
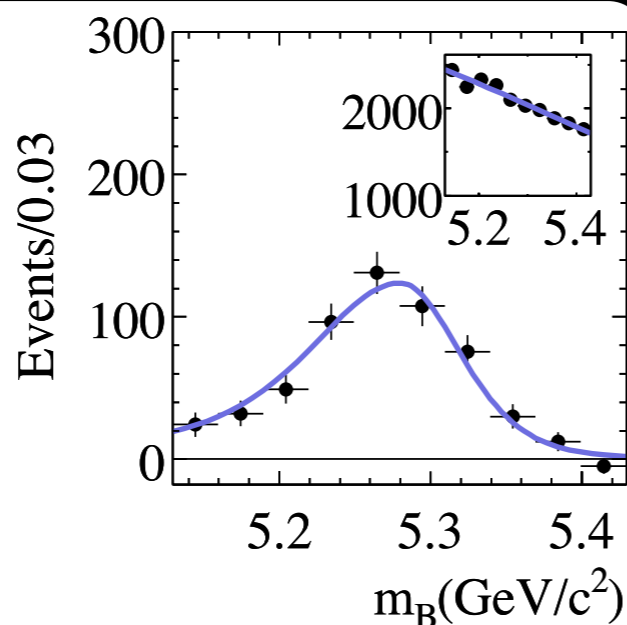
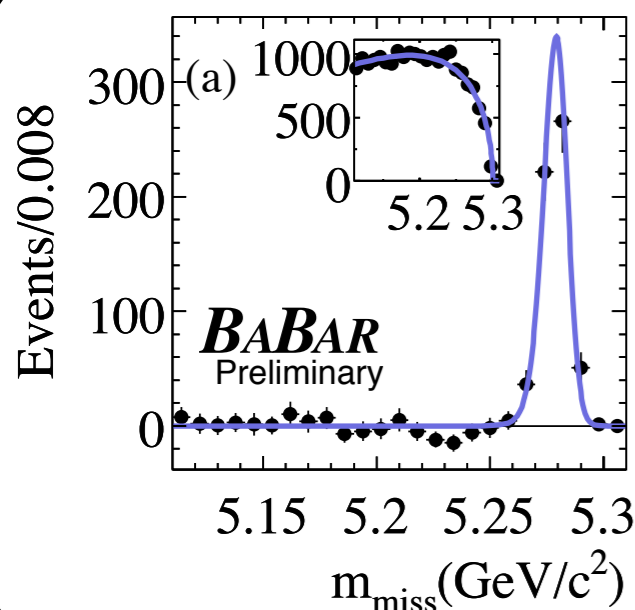
¹Pivk and LeDiberder, NIM Phys. Res., Sect. A 555, 356 (2005).



$$B^0 \rightarrow \pi^0 K_S^0$$

arXiv:0809.1174 (submitted to PRD)

- No tracks from B decay vertex!
- 60% of signal B candidates make hits in inner silicon tracker (SVT).
- Obtain Δt in these events w/ constraints on average interaction point and B lifetime.
- Δt resolution comparable to other modes (~ 1 ps).
- All 556 ± 32 signal events constrain C .



$$S = 0.55 \pm 0.20 \pm 0.03$$

$$C = 0.13 \pm 0.13 \pm 0.03$$

Results Summary

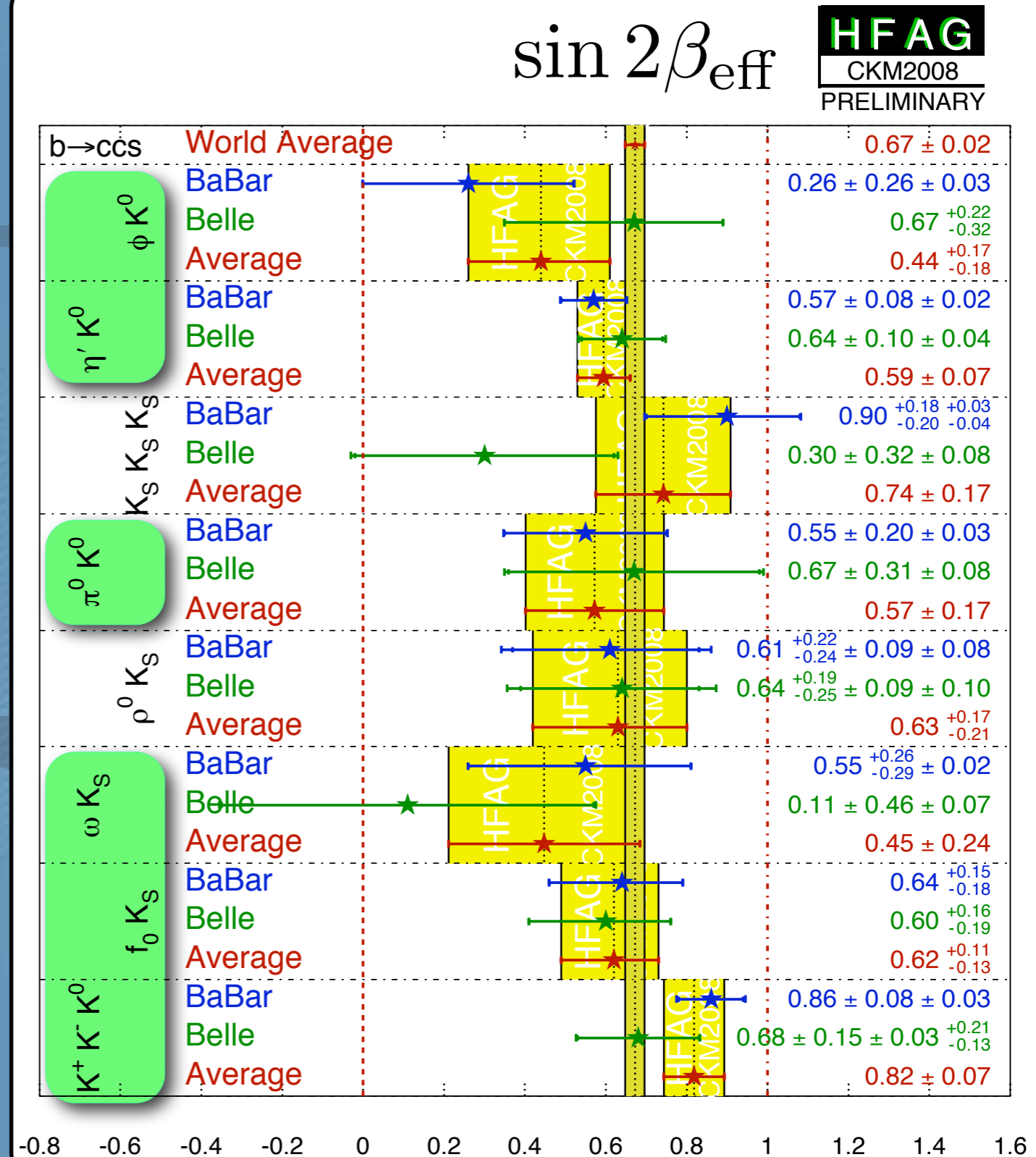
$$-\eta_f S_f$$

$$C_f$$

$\eta' K^0$	$0.55 \pm 0.08 \pm 0.02$	$-0.09 \pm 0.06 \pm 0.02$
$\eta' K_S^0$	$0.53 \pm 0.08 \pm 0.02$	$-0.11 \pm 0.06 \pm 0.02$
$\eta' K_L^0$	$0.64 \pm 0.20 \pm 0.03$	$0.05 \pm 0.15 \pm 0.03$

- In 2005, naive average of ΔS was 3.7σ from zero.
- Discrepancy has shrunk to 0.7σ .
- C measurements consistent w/ 0.
- No indication of new physics.

- The B-factories have exceeded all expectations!
- CKM phase is the dominant source of CP violation!



New physics flavor puzzle at the LHC

- New physics (NP) expected at energy scale $\Lambda \sim 1$ TeV,
 - to prevent divergence of the Higgs mass.
- New physics flavor puzzle:

How does NP, expected to have generic flavor structure, maintain the observed flavor structure of the Standard Model?
- Solution: NP follows principle of minimal flavor violation¹ (MFV);
 - i.e., the SM Yukawa couplings are the dominant source of flavor violation.
- It may be possible to experimentally *exclude* MFV at the LHC; consider this fortunate scenario²:
 - \tilde{t} is next-to-lightest SUSY particle.
 - $\tilde{t} \rightarrow \chi_1^0 b$ is kinematically forbidden.
 - Large decay rate for $\tilde{t} \rightarrow \chi_1^0 c$ can exclude MFV.

¹ D'Ambrosio, Giudice, Isidori, Strumia, hep-ph/0207036;
Cirigliano, Grinstein, Isidori, Wise, hep-ph/0507001, hep-ph/0608123;
Isidori, Mescia, Paradisi, Smith, Trine, hep-ph/0604074;
Nikolidakis, Smith, arXiv:0710.3129;

² Hiller and Nir, arXiv:0802.0916
Nir, arXiv:0708.1872

Summary

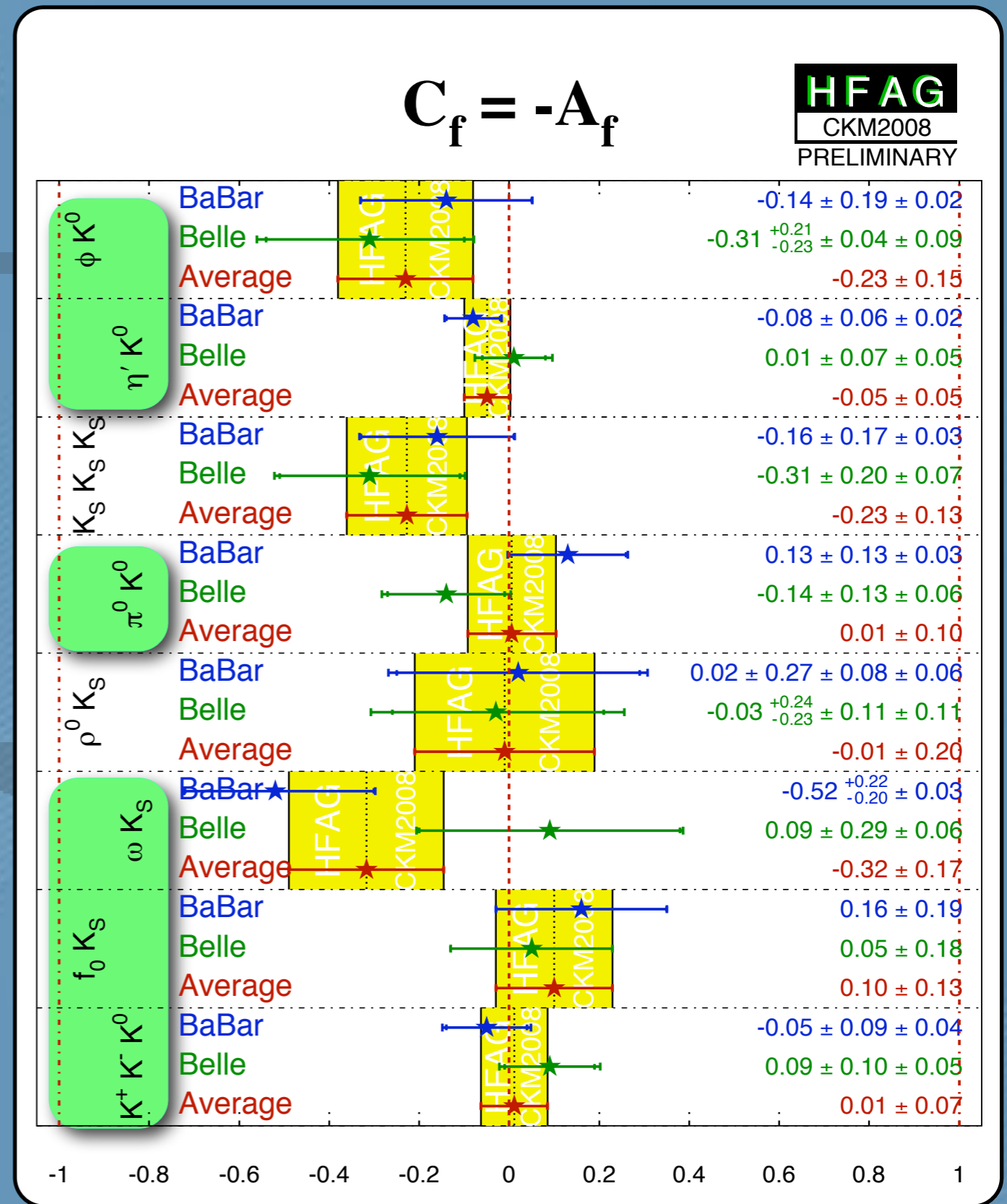
- $\sin 2\beta$ measured in $b \rightarrow c\bar{c}s$ decays such as $B^0 \rightarrow J/\psi K^0$.
 - $\sin 2\beta_{\text{eff}}$ measured in $b \rightarrow q\bar{q}s$ penguin decays such as $B^0 \rightarrow \eta' K^0$.
 - In Standard Model $\sin 2\beta_{\text{eff}} \simeq \sin 2\beta$.
 - $\sin 2\beta_{\text{eff}}$ sensitive to non-SM particles in the loop.
-
- In 2005, there was large discrepancy ($\sim 4\sigma$) between $\sin 2\beta$ and $\sin 2\beta_{\text{eff}}$.
 - Current measurements differ by only $\sim 1\sigma$.
 - CKM phase is the sole source of CP violation.
-
- New physics flavor puzzle:
If new physics is there at $\Lambda \sim 1$ TeV, why do we find no evidence at B-factories?
 - At LHC, Minimal Flavor Violation will be easier to exclude than confirm.



Extra slides

Summary

	$-\eta_f S_f$	C_f
$\eta' K^0$	$0.55 \pm 0.08 \pm 0.02$	$-0.09 \pm 0.06 \pm 0.02$
$\eta' K_S^0$	$0.53 \pm 0.08 \pm 0.02$	$-0.11 \pm 0.06 \pm 0.02$
$\eta' K_L^0$	$0.64 \pm 0.20 \pm 0.03$	$0.05 \pm 0.15 \pm 0.03$



A Tale of Two Bases

- The Standard Model describes Nature in terms of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry.
- $SU(2)_L \times U(1)_Y$ sub-group is spontaneously broken by Higgs mechanism putting Yukawa term in Lagrangian:

$$\mathcal{L}_Y = -\lambda_{ij}^d \bar{Q}_i^L \phi d_j^R - \lambda_{ij}^u \bar{Q}_i^L \phi_c u_j^R + \text{h.c.},$$

- Change from flavor basis into new basis that diagonalizes $\lambda^{u,d}$:

$$\mathcal{L}'_Y = -\frac{v}{\sqrt{2}} \lambda_{ii}^{d'} \bar{d}_i^{L'} d_i^{R'} - \frac{v}{\sqrt{2}} \lambda_{ii}^{u'} \bar{u}_i^{L'} u_i^{R'} + \text{h.c.}$$

- This term describes quarks with masses:

$$m_{ii}^d = \frac{v}{\sqrt{2}} \lambda_{ii}^{d'}, \quad m_{ii}^u = \frac{v}{\sqrt{2}} \lambda_{ii}^{u'}$$

- Quark fields in this mass basis:

$$u_i^{L'} = U_{ij}^u u_i^L, \quad d_i^{L'} = U_{ij}^d d_i^L$$

$\lambda^{u,d}$	3x3 complex matrices
\bar{Q}^L	quark doublet
u^R, d^R	quark singlets
ϕ	Higgs doublet
i, j	flavor indices
$\phi_c \equiv -i\tau_2 \phi^*$	

$B\bar{B}$ Time Evolution

- Mass and flavor eigenstates differ:

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

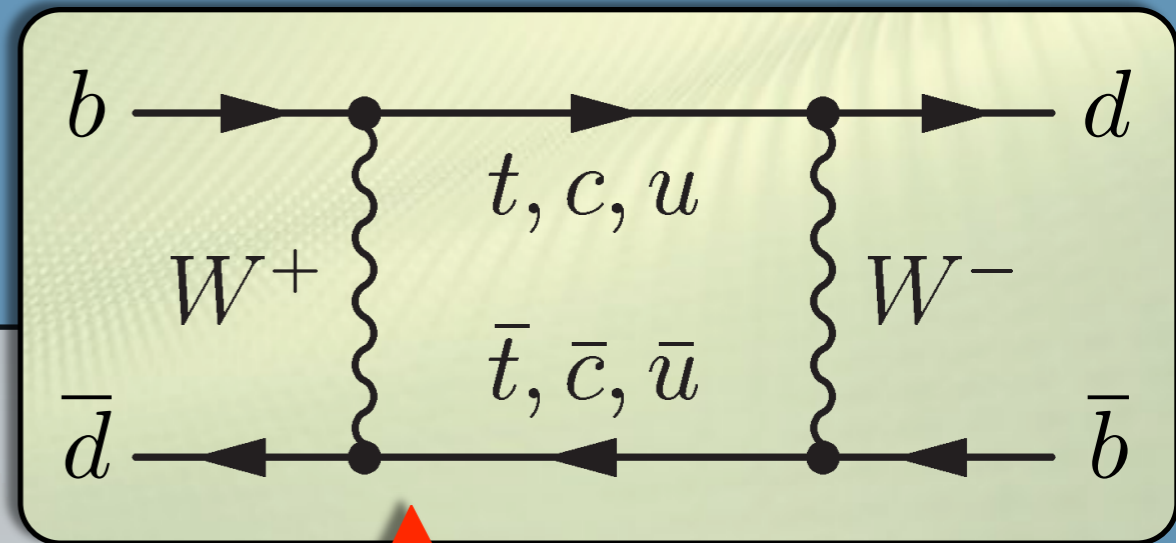
- Note $\Delta\Gamma_B \ll \Delta m_B$ (in K system $\Delta\Gamma_K \simeq 2\Delta m_K$).
- Neutral B's mix via second-order weak process (box diagram).
- Write time-dependent Schrodinger equation for two-state system with mixing and decay; write state of particle produced as B^0 at $t = 0$ in terms of mass states and eigenvalues of Hamiltonian $\lambda_{L,H}$:

$$|B_{\text{phys}}^0(t)\rangle = \frac{1}{2p} [e^{-i\lambda_H t}|B_H\rangle + e^{-i\lambda_L t}|B_L\rangle]$$

- In terms of $M = (M_H + M_L)/2$ and $\Gamma \simeq \Gamma_H \simeq \Gamma_L$:

$$|B_{\text{phys}}^0(t)\rangle = e^{-iMt}e^{-\Gamma t/2} \left[\cos\left(\frac{1}{2}\Delta m_B t\right) |B^0\rangle + i\frac{q}{p} \sin\left(\frac{1}{2}\Delta m_B t\right) |\bar{B}^0\rangle \right]$$

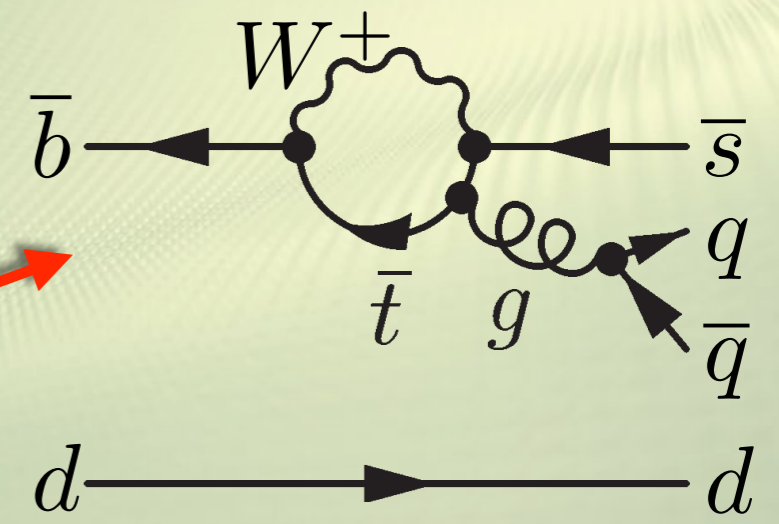
- Similarly for $|\bar{B}_{\text{phys}}^0(t)\rangle$.



$$\Delta m_B \equiv m_H - m_L$$

$$\Delta\Gamma_B \equiv \Gamma_H - \Gamma_L$$

$\sin 2\beta_{\text{eff}}$ from $b \rightarrow q\bar{q}s$ penguin decays



- $b \rightarrow q\bar{q}s$ decays occur via loop processes in SM.
- $b \rightarrow s$ penguins also sensitive to $\sin 2\beta$ but with theoretical uncertainty from tree and Cabibbo-suppressed penguin pollution:
- In terms of tree (T) and penguin (P_q , $q = u, c, t$) contributions:

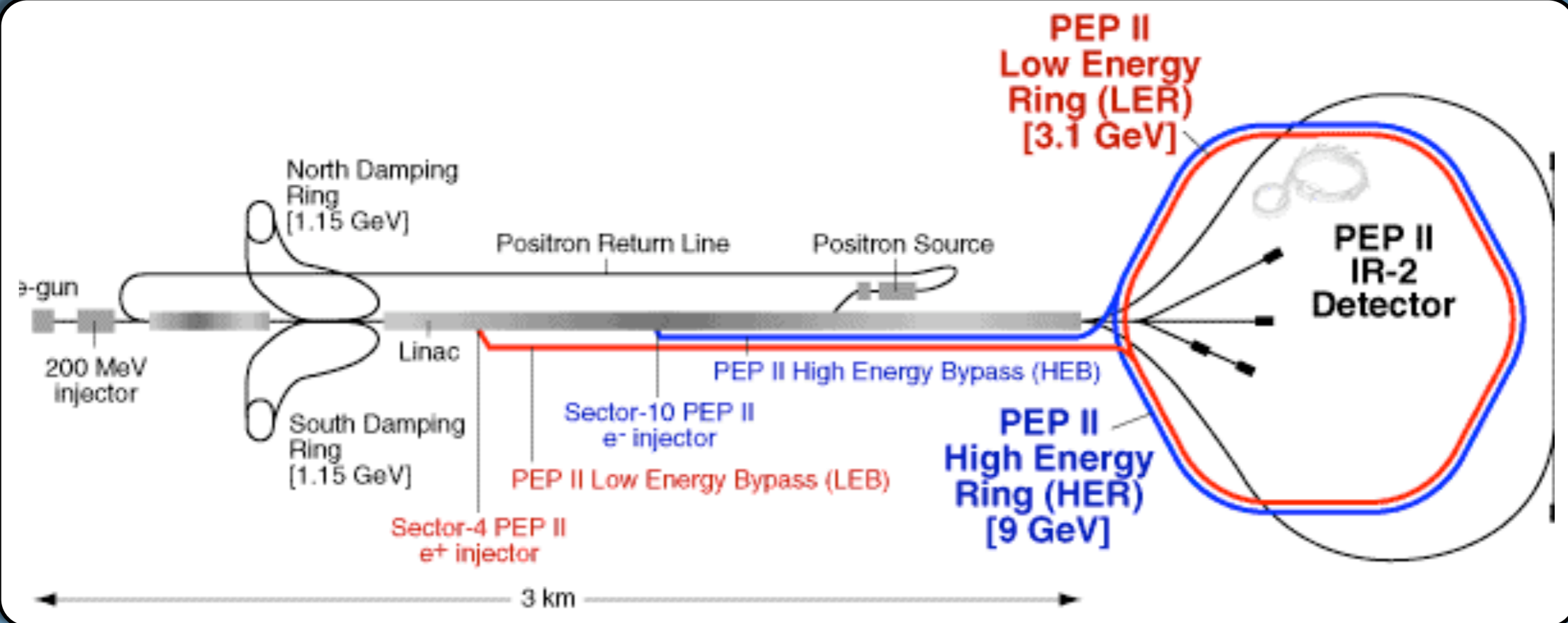
$$A_{\eta' K^0} = P_t V_{tb} V_{ts}^* + P_c V_{cb} V_{cs}^* + (P_u + T) V_{ub} V_{us}^*$$

- Unitarity gives

$$\begin{aligned} A_{\eta' K^0} &= (P_c - P_t) V_{cb} V_{cs}^* + (P_u - P_t + T) V_{ub} V_{us}^* \\ &\simeq \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^4) \end{aligned}$$

- The first term, with same weak phase as $b \rightarrow c\bar{c}s$ decays, dominates.
- However, $|\bar{A}_{\eta' K^0}|/|A_{\eta' K^0}| - 1 = \mathcal{O}(\lambda^2)$, so

$$S_{\eta' K^0} = \sin 2\beta_{\text{eff}} \simeq \sin 2\beta$$



Maximum likelihood method

- Consider set of N measurements of a quantity x distributed according to a *probability density function* (PDF) $\mathcal{P}(x, \alpha)$ where α is a set of n parameters $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$.

$$\mathcal{L} = \prod_i^N \mathcal{P}(x_i, \alpha)$$

- The likelihood function is defined :
- Given the data (x_i) , the likelihood gives the relative probability for values of parameters α . The maximum of \mathcal{L} over parameter space α gives an unbiased estimate of α .
- For large N , $\mathcal{L}(\alpha_i)$ is Gaussian near maximum $\mathcal{L}_{\max} \equiv \mathcal{L}(\hat{\alpha}_i)$:

$$\mathcal{L}(\alpha_i) = \mathcal{L}_{\max} e^{-\frac{(\alpha_i - \hat{\alpha}_i)^2}{2\sigma^2}}$$

- Statistical uncertainty σ_i on α_i is $\frac{1}{\sigma_i^2} = \frac{\partial^2 \ln \mathcal{L}}{\partial \alpha_i^2}$.
- We compute the significance (\mathcal{S}) of a fit result (\mathcal{L}_{\max}) relative to some other hypothesis, e.g. the zero signal hypothesis (\mathcal{L}_0), with the likelihood ratio test: $\mathcal{S} = -2 \ln \frac{\mathcal{L}_{\max}}{\mathcal{L}_0}$.

ML fit specifics

- If N is a random variable, construct *extended* likelihood with factor for Poisson probability of making N measurements when expecting ν .
- We use multiple observables so $x \rightarrow \mathbf{x}$, where $\mathbf{x} = \{x_1, x_2, \dots, x_m\}$.
- The PDF $\mathcal{P}(\mathbf{x}, \boldsymbol{\alpha})$ is a composite with each part corresponding to a component of the data, such as signal or background.
- Correlations between variables of \mathbf{x} are low ($<5\%$), so we factorize the PDF for each fit component j : $\mathcal{P}_j(\mathbf{x}) = \mathcal{P}_j(x_1)\mathcal{P}_j(x_2)\dots\mathcal{P}_j(x_m)$.
- In practice, \mathcal{L} is very small and computationally difficult, so we minimize $-2 \ln \mathcal{L}$ instead of maximizing \mathcal{L} .
- The likelihood for N events with signal (sig) and background (bkg) components:

$$\mathcal{L} = \frac{e^{-(\nu_{\text{sig}} + \nu_{\text{bkg}})}}{N!} \prod_i^N \nu_{\text{sig}} \mathcal{P}_{\text{sig}}(\mathbf{x}_i, \boldsymbol{\alpha}_{\text{sig}}) + \nu_{\text{bkg}} \mathcal{P}_{\text{bkg}}(\mathbf{x}_i, \boldsymbol{\alpha}_{\text{bkg}})$$

where ν_{sig} and ν_{bkg} (the estimators of the sig and bkg event yields) are free to vary in the fit.

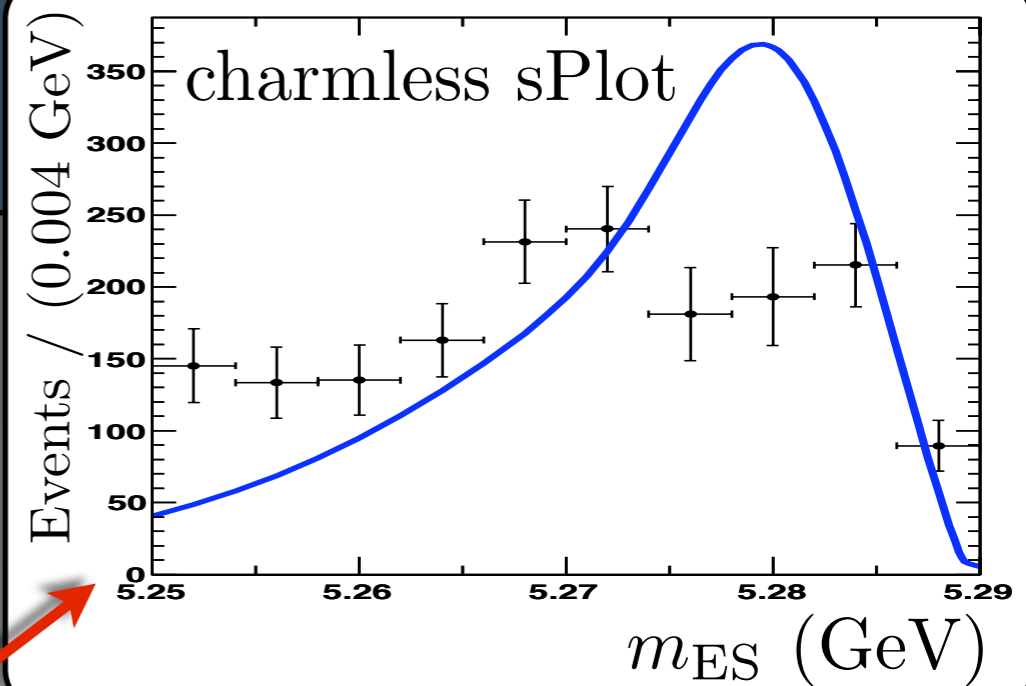
KL cut optimization

Table A.1: Results from cut optimization study for $\eta'_{\eta\pi\pi}K_L^0$. The “Very Tight” cut values maximize signal significance ($S/\sqrt{S+B}$). The “Loose” cut values minimize the errors on S and C from toy MC studies and are used in this analysis. We report cut values, events entering the Run1-6 fit, signal efficiency, expected Run1-6 signal yield, mean of the S and C error distributions for embedded toys with Run1-6 statistics, and blind fit values to run1-6 data as a final crosscheck.

	Very Tight	Tight	Loose	Very Loose
NN output Cut	0.50	0.40	0.30	0.20
$P_{\text{miss}}^{\text{proj}}$ Cut	−0.46	−0.60	−0.70	−0.80
$\cos\theta_{P_{\text{miss}}}$ Cut	0.93	0.94	0.95	0.96
Events to Fit	6253	8826	12085	14992
MC ϵ (%)	15.2	17.6	19.8	21.0
Expected nSig	249	310	353	375
S error	0.310 ± 0.004	0.273 ± 0.003	0.257 ± 0.003	0.262 ± 0.003
C error	0.222 ± 0.002	0.198 ± 0.002	0.191 ± 0.002	0.190 ± 0.001
blind S	$−0.76 \pm 0.26$	$−0.68 \pm 0.24$	$−0.68 \pm 0.22$	$−0.59 \pm 0.22$
blind C	0.09 ± 0.22	0.05 ± 0.21	0.04 ± 0.19	0.05 ± 0.19

Charm $B\bar{B}$ background

- Typically, charm events are absorbed into continuum background yield in ML fit.
- In previous analyses, charmless yield floated 1.6 -- 5 times higher than expected.
- Bias on charmless yield was negative.
- In mode $e\tau\mu K^+$, projection of charmless events onto m_{ES} shows contamination from non-peaking events -- charm or qq ?

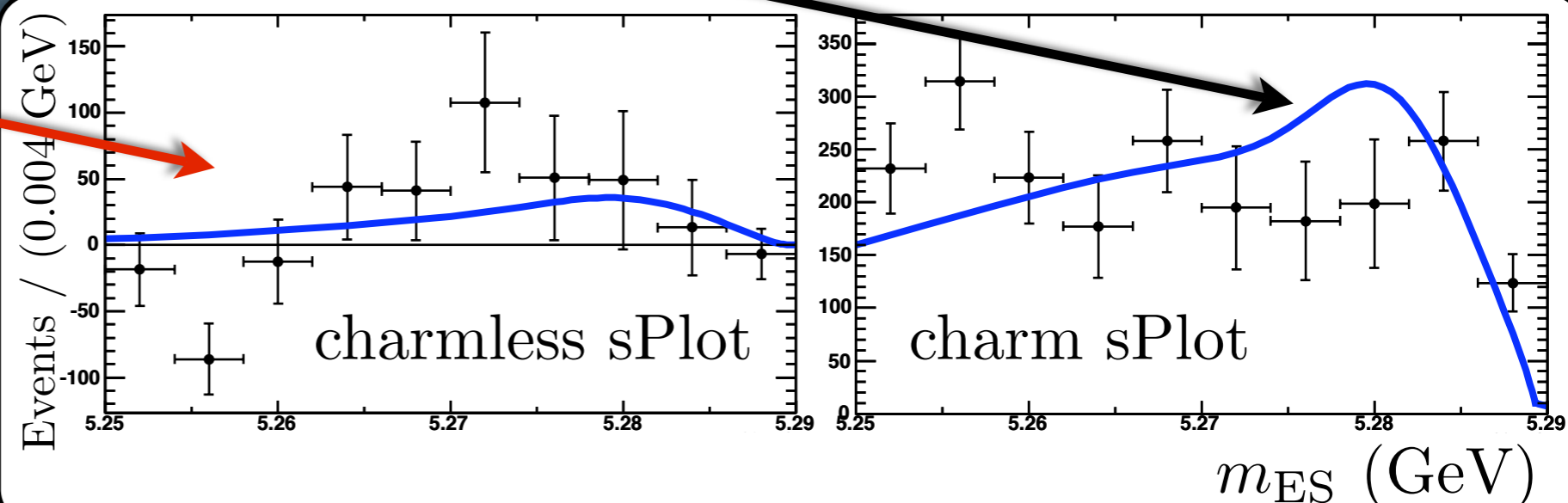


sPlots

Accumulate probability to be signal (computed without plotted variable) in bins of variable of interest. Overlay normalized PDF.

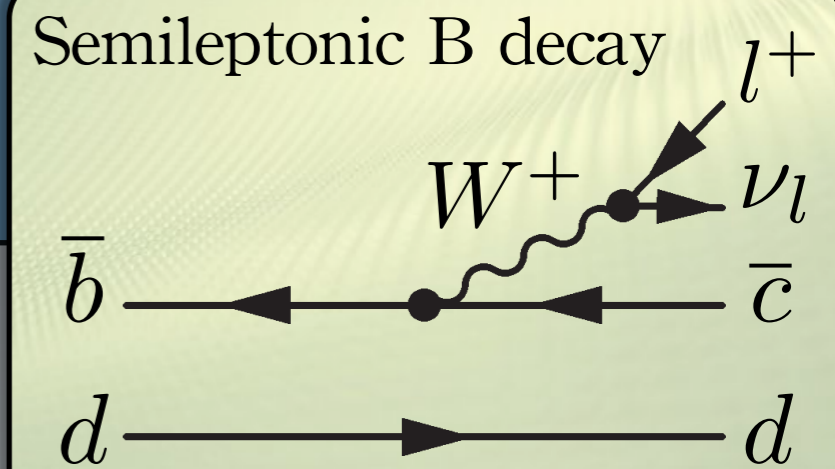
- With charm component in ML fit projections look more reasonable.
- Charm and charmless yields float to reasonable values in ML fit.

Charm PDF is $q\bar{q}$ -like with small peaking component.



Tagging algorithm

- Neural-network-based algorithm assigns each B_{tag} candidate to 1 of 6 categories.
- Category is determined by continuous output of algorithm (and lepton in the B_{tag} final state).
- Cleanest tagging from semileptonic decays, such as $B^0 \rightarrow D^{*-} l^+ \nu_l$.
- Clean tagging from $b \rightarrow c \rightarrow s$ decays, such as $B^0 \rightarrow D^{*-} \rho^+$, $D^{*-} \rightarrow \bar{D}^0 \pi^-$, $\bar{D}^0 \rightarrow K^+ \pi^-$



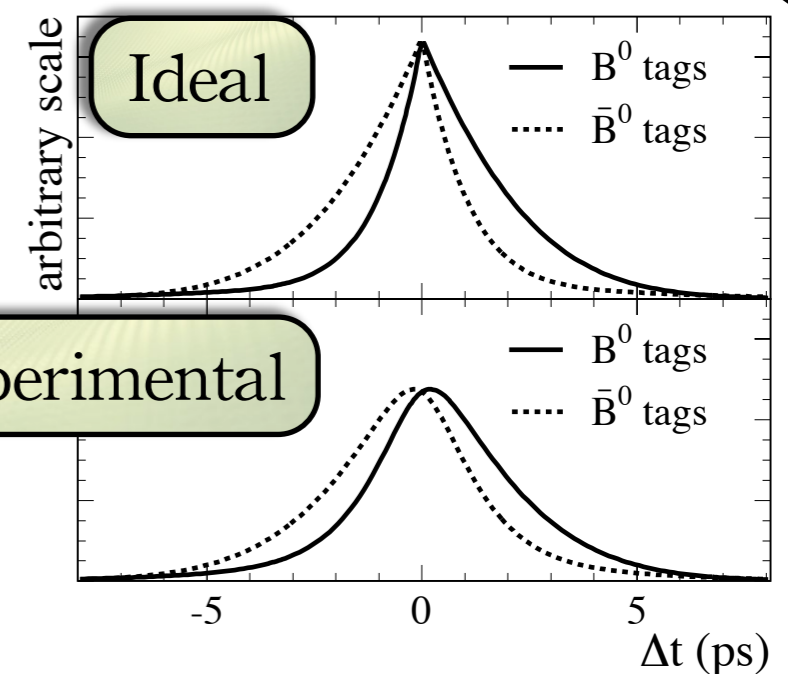
Δt resolution function

- Sum of 3 Gaussians (*core*, *tail*, and *outlier*).
- The means and widths of core, tail Gaussians are scaled by uncertainty on Δt ($\sigma_{\Delta t}$).

$$\mathcal{R}(\Delta t, \sigma_{\Delta t}) = (1 - f_{\text{tail}} - f_{\text{out}})G(\Delta t, \mu_{\text{core}}\sigma_{\Delta t}, \sigma_{\text{core}}\sigma_{\Delta t}) + f_{\text{tail}}G(\Delta t, \mu_{\text{tail}}\sigma_{\Delta t}, \sigma_{\text{tail}}\sigma_{\Delta t}) + f_{\text{out}}G(\Delta t, \mu_{\text{out}}, \sigma_{\text{out}})$$

$G(x, \mu, \sigma)$ is Gaussian in x of mean μ and width σ .

- Parameters differ b/w tagging categories.



Validating fit uncertainties

Table 1: Mean values of the error distributions for S and C and the RMS of the S and C distributions for each sub-mode from 140-550 embedded toy MC experiments. (The number of experiments is the maximum without oversampling the signal MC.) We also report the averaged sub-mode results and the results from 175 toy experiments for the simultaneous fit. All results are reported in units of 10^{-3} .

Final state	Mean of S Error	RMS of S Error	RMS of S dist.	Mean of C Error	RMS of C Error	RMS of C dist.
$\eta'_{\eta\pi\pi} K_S^0$	156 ± 1	14 ± 1	167 ± 8	115 ± 1	5 ± 1	121 ± 6
$\eta'_{\rho\gamma} K_S^0$	112 ± 1	8 ± 1	114 ± 7	88 ± 1	3 ± 1	97 ± 6
$\eta'_{\eta\pi\pi} K_{S00}^0$	381 ± 5	75 ± 5	481 ± 34	267 ± 2	31 ± 2	280 ± 20
$\eta'_{\rho\gamma} K_{S00}^0$	325 ± 3	53 ± 3	357 ± 22	258 ± 2	36 ± 2	267 ± 16
$\eta'_{5\pi} K_S^0$	246 ± 3	38 ± 3	280 ± 20	180 ± 1	14 ± 1	192 ± 14
$\eta'_{\eta\pi\pi} K_L^0$	271 ± 2	32 ± 2	278 ± 18	199 ± 1	15 ± 1	201 ± 13
$\eta'_{5\pi} K_L^0$	358 ± 6	70 ± 6	416 ± 35	265 ± 3	32 ± 3	263 ± 22
Weighted						
Average :	76 ± 0	7 ± 0	79 ± 5	57 ± 0	2 ± 0	61 ± 4
Simultaneous Fit:	75 ± 0	3 ± 0	75 ± 6	58 ± 0	2 ± 0	61 ± 6

Systematic Errors

- Beamspace position: Fit MC with reasonable variations in y , σ_y .
- SVT alignment: Fit MC with reasonable SVT mis-alignment.
- Tag-side interference: With toy MC estimate effect of interference b/w CKM-suppressed $b\bar{b} \rightarrow u\bar{c} d\bar{b}$ and favored $b\bar{b} \rightarrow u\bar{c} d\bar{d}$ amplitudes
- Misreconstructed Signal: Depending on sub-mode, in 1-4% of reconstructed signal events we swap track with rest of event. We embed various concentrations of *self-crossfeed* to estimate effect on DT and S/C.
- Fit bias: Take uncertainty on fit bias as systematic.
- PDF parameterization: Vary fixed params by amounts found in studies of data control samples (B_{flav} , $B \rightarrow \pi D$, $\eta' \rightarrow K^+ \pi^-$) take DS/DC as systematic.
- BB background: Vary CP params for chls, charm components; vary fixed charmless yield by $\pm 20\%$.
- Signal DT parameterization : By comparing toy MC studies performed with DT params (calR and tagging) from signal and B_{flav} MC, we estimate effects due to differences between signal and B_{flav} data.

Source of error	$\sigma(S)$	$\sigma(C)$
Beam position/size	0.002	0.001
SVT alignment	+0.002 -0.001	+0.003 -0.002
Tag-side interference	0.001	0.015
Self-crossfeed	0.006	0.003
Fit Bias	0.006	0.006
PDF Shapes	0.005	0.009
$B\bar{B}$ Background	0.008	0.004
Signal Δt Shape	0.009	0.010
Total	0.016	0.022

Crosschecks

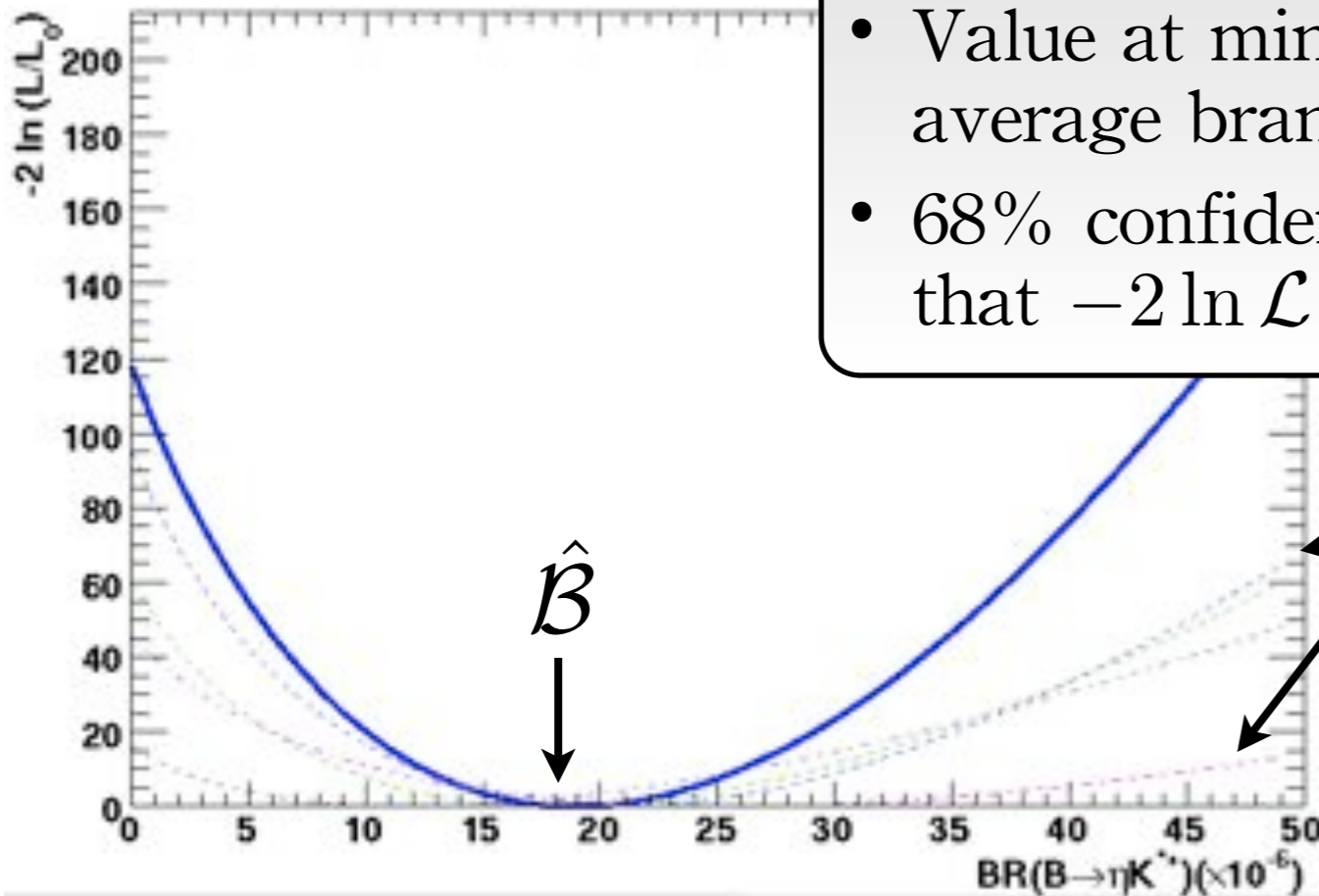
- Including PDFs for m_η , m_ρ , θ_{dec}^ρ does not improve precision on S and C.
- Fit is stable without PDFs for (one-at-a-time). m_{ES} , ΔE , and \mathcal{F}
- S and C are consistent across tagging categories.
- Δt model is reasonable:
 $\tau = 1.53 \pm 0.07$ (PDG is 1.5 ± 0.01)
- S and C in charged modes consistent with zero.

Tag Cat	S	C
Lepton	-0.670 ± 0.134	0.122 ± 0.105
Kaon1	-0.315 ± 0.146	0.132 ± 0.107
Kaon2	-0.569 ± 0.150	0.297 ± 0.117
KaonPion	-0.246 ± 0.255	0.080 ± 0.185
Pions	-0.205 ± 0.365	0.263 ± 0.257
Other	-1.414 ± 1.010	0.624 ± 0.710
Weighted Average	-0.492 ± 0.076	0.174 ± 0.057
Nominal Result	-0.481 ± 0.078	0.174 ± 0.058

	Nominal Fit	All-Variable Fit
$\eta'_{\eta\pi\pi} K_S^0$		
S	-0.622 ± 0.166	-0.629 ± 0.165
C	0.231 ± 0.110	0.217 ± 0.109
$\eta'_{\rho\gamma} K_S^0$		
S	-0.370 ± 0.117	-0.373 ± 0.116
C	0.217 ± 0.090	0.199 ± 0.086
$\eta'_{\eta\pi\pi} K_{S00}^0$		
S	-0.402 ± 0.347	-0.390 ± 0.342
C	0.232 ± 0.301	0.266 ± 0.301
$\eta'_{\rho\gamma} K_{S00}^0$		
S	-0.179 ± 0.331	-0.518 ± 0.336
C	0.029 ± 0.261	0.107 ± 0.263
$\eta'_{5\pi} K_S^0$		
S	-0.650 ± 0.261	-0.671 ± 0.260
C	0.036 ± 0.199	0.039 ± 0.198
$\eta'_{\eta\pi\pi} K_L^0$		
S	-0.684 ± 0.223	-0.692 ± 0.218
C	0.036 ± 0.194	0.033 ± 0.184
combined S	-0.482 ± 0.079	-0.507 ± 0.078
combined C	0.179 ± 0.059	0.171 ± 0.058

Fit vars	$m_{\text{ES}}, \Delta E, \mathcal{F}, \Delta t$	$\Delta E, \mathcal{F}, \Delta t$	$m_{\text{ES}}, \mathcal{F}, \Delta t$	$m_{\text{ES}}, \Delta E, \Delta t$
S	-0.481 ± 0.078	-0.471 ± 0.080	-0.512 ± 0.077	-0.535 ± 0.082
C	0.174 ± 0.058	0.159 ± 0.062	0.164 ± 0.059	0.165 ± 0.063
Signal Yields				
$\eta'_{\eta\pi\pi} K_S^0$	468.8 ± 23.5	465.4 ± 26.2	461.8 ± 23.5	475.1 ± 24.9
$\eta'_{\rho\gamma} K_S^0$	999.0 ± 39.6	997.0 ± 47.5	966.7 ± 46.9	1007.2 ± 45.4
$\eta'_{\eta\pi\pi} K_{S00}^0$	104.3 ± 13.0	107.4 ± 15.8	104.3 ± 12.7	111.3 ± 18.0
$\eta'_{\rho\gamma} K_{S00}^0$	201.9 ± 27.4	288.7 ± 44.6	196.2 ± 29.7	201.9 ± 44.9
$\eta'_{5\pi} K_S^0$	170.5 ± 14.1	177.4 ± 15.8	175.9 ± 23.1	172.3 ± 14.9
$\eta'_{\eta\pi\pi} K_L^0$	331.5 ± 31.4	334.9 ± 31.5	334.6 ± 29.1	334.6 ± 31.5
$\eta'_{5\pi} K_L^0$	163.9 ± 21.8	160.9 ± 21.6	160.8 ± 19.8	160.7 ± 21.6

Averaging results



- In LMR:
- Obtain $-2 \ln \mathcal{L}_f$ as function of ηK_1^* branching fraction (\mathcal{B}) for each sub-mode.
- Convolve $-2 \ln \mathcal{L}_f(\mathcal{B})$ w/ Gaussian of with σ_{syst} .
- Value at minimum of $\sum -2 \ln \mathcal{L}_f(\mathcal{B})$ is average branching fraction ($\hat{\mathcal{B}}$).
- 68% confidence interval on $\hat{\mathcal{B}}$ is σ such that $-2 \ln \mathcal{L}(\hat{\mathcal{B}} \pm \sigma) = 1$.

Dashed sub-mode curves

- In HMR:
 - ηK_0^* and ηK_2^* are highly correlated (40%).
 - Combining with $-2 \ln \mathcal{L}_f(\mathcal{B})$ curves neglects correlations.
 - We average with simultaneous fits (as in $\eta' K^0$ analysis), one fit for 2 neutral modes, one for 4 charged modes.